Insurance in the Age of Wisdom and Foolishness: A Tale of Healthy City*

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Abstract

We model the political economy for the provision of public long-term care services in an economy where the demand for privately-provided and publicly-provided long-term care services are jointly determined with the demand for long-term care insurance. We make use of Salop's circular-city approach with randomly determined search costs and free entry in the supply of private long-term care services. We introduce a central supplier (government) for which the travel cost is also random. We show that in a market without insurance, it is possible to have a multitude of consistent market equilibria such as having all the market catered by the private sector, all the market serviced by the government, partial government-provision of services, and perfectly segmented market whereby agents that have relatively low (resp. high) travel cost to the government opt for government-provided (resp. privately-provided) services. The paper concludes with a number of comparative static results and recommendation to public policy makers.

Keywords: Longevity risk, Supply and demand of long-term care services and insurance; Salop's circular city; Government programs; Informal care.

JEL-Classification: G02 · G12 · C14

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> *"It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to Heaven, we were all going direct the other way - in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only."*
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> Charles Dickens

1 Introduction

One of the biggest sociological and financial challenges that await OECD countries (OECD, 2011) in the coming decades, aside from global warming and ecological changes, are demographic in nature. As rich Western countries see the share of their elderly population growing older, their demography will soon reach a point where there will be only two working individuals per retiree. By 2050, it is projected that countries such as Germany, Italy, Japan, and South Korea are projected to have a ratio of working-to-retired close to 1.5.[1] As the ratio of working-to-retired declines rapidly, one major challenge for public policy makers who want to stem the rising so-called *grey tsunami*[2] resides in the design and the financing of a system that will respond to the greater need for health services in those older ages. This is particularly true in the case of long-term care services for which the need will likely increase as the Western world's population grows older (Lakdawalla and Philipson, 2002), but not necessarily healthier. The provision of long-term care services is only one of many problems that OECD countries must face with respect to their growing elderly population. In contrast to medical care for the elderly and retirement planing for which great advances have been made, the great majority of OECD countries are still looking for the best way to provide and finance the need for long-term care services.

The theoretical model we will develop finds a clear application in the long-term care market, with supplier of services competing for clients, governments offering services, as well as insurers catering to a (small) portion of the economy.

We first present in the next section one application of the economic situation that our model seeks to explain: The growing need of long-term care services and insurance that is

[1] https://yaleglobal.yale.edu/content/number-workers-retiree-declines-worldwide, last visited on 12 March 2019.

[2] The term *grey tsunami*, which refers to the rising proportion of the population at older ages, is being more and more criticized. In particular, Barusch (2013) considers this term to be 'a nasty metaphor for older adults'. In the search of a new metaphor to refer to the rising political, economic, and sociological clout that the older tranche of the population will have, Barusch (2013) concedes that 'so far no one has offered a compelling substitute for the tsunami – something gentle, expansive, and enduring, like the autumn sun', perhaps something like the 'argyrocracy elite'.

predicted to befall OECD countries as their cope with a growing elderly population. This long primer on the problem will cover the states of long-term care services (Section 2.1), the supply of long-term care services (Section 2.2), the role of governments (Section 2.3), and the role of families (Section 2.4). In Section 3, we present the model we propose to use to analyze the problem at hand. This model introduces a multidimensional Salop circular-city with many levels of consumer heterogeneity. We then solve for the market equilibrium in Section 4 by examining supply side reactions of social programs provided by the government, and the role of families in providing long-term care services to their ailing elderly parents. Section 5 introduces insurance contracts to cover the potential cost of private-provided services, and discusses the implication of such contracts on equilibrium. We discuss public policy implications in Section 6, and conclude and provide avenues of future research in Section 7

2 A Primer on long-term care

Long-term care is defined as the care for elderly individuals over a prolonged period of time. This care is provided in the form of support with activities of daily living (such as bathing, dressing, eating, getting in and out of bed, grooming, and continence) or with instrumental activities of daily living (which include preparing meals, cleaning, doing the laundry, taking medication, getting to places beyond walking distance, shopping, managing money, and using the telephone or the Internet). Long-term care is thus related to the loss of autonomy brought on by old age.[3] It is important to distinguish upstream (acute care or rehabilitation) from downstream (help with activities of daily living) services since the former is generally taken care of by health professionals, whereas the latter is often provided by relatively unskilled workers and family members.

LTC should be distinguished from illness, disability, and handicap, which can affect younger individuals. Because needing LTC is not the same as having a disability, LTC insurance is not the same as disability insurance. Disability insurance is more targeted towards the working age population whereas LTC insurance is targeted towards the retired or soon-to-be-retired population.

Financing LTC services raises many challenges since LTC is becoming an increasingly important problem for all developed countries. According to OECD (2011), the population aged 80 and over is expected to represent 10% of the developed world's population by 2050. That age bracket represented only 4% of the rich world's total population in 2010. The over 80 age range is the fastest growing age group in the developed world. The fact that the

[3] The National Institute on Aging in the United States defined long-term care as "a variety of services designed to meet a person's health or personal care needs during a short or long period of time ... when they can no longer perform everyday activities on their own"; https://www.nia.nih.gov/health/what-long-term-care, last visited on 12 March 2019.

proportion of the population that is elderly increases would not be a problem in itself if the up-and-coming population aged 80 in 2050 were as healthy as the population aged 70 in 2010. The challenge for the provision and the financing of LTC services is that the average number of years during which LTC services will be needed may actually increase if the population grows older but not healthier, or that the types of services needed, sought, and/or covered by the public system or private insurers in the future changes.

Some studies, including those of Brown and Finkelstein (2008) and Brown and Finkelstein (2009), argue that private long-term care insurance contracts are expensive because of important loading factors. Brown and Finkelstein (2007) show, however, that loads on LTC insurance (that is, the ratio of the premium to the expected present value of the benefits) are not particularly high; at least not so high as to lead rich retirees to prefer using their private savings as a form of self-insurance rather than purchasing LTC insurance. In addition, as reported in Davidoff (2013), the loading factor for women is essentially zero. Other studies, such as Sloan and Norton (1997) point to the existence of important asymmetric information problems (both moral hazard and adverse selection) which induce insurers to restrict coverage.

2.1 The state of LTC services and insurance

Long-term care is a storm in waiting. Every OECD country is facing a rapidly aging population which is projected to be in more and more needs for long-term care services. For instance in Canada, the Conference Board, a non-partisan think tank, anticipates that the country will need an additional 199,000 long-term beds by 2035.[4] Give that the number of beds available in 2018 is estimated to be 255,000, the Conference Board anticipates a growth rate in the number of beds of 3.5% per year. In contrast, Figure 1 the average annual growth in the proportion of 65 year olds needing institutional care has been approximately 1% in the following set of OECD countries.

2.2 Supply of LTC facilities and LTC insurance characteristics

Figure 2 provides the proportion of the population receiving long-term care services in 2000 and 2013. We note that in most country, the proportion of the population receiving long-term care services has increased over these 13 years. On average, for the 21 OECD countries for which there are reliable value, the proportion of the population receiving long-term care service has increased from 1.87% to 2.33% (a 25% increase)

Despite the clear and present investment in long-term care facilities, private long-term care insurance is not very developed in any OECD country. Figure 3 shows that in OECD

[4] See also Blomqvist and Busby (2014) and Adams and Vanin (2016).

countries, the private LTC insurance market represents between 1% and 2% of total long-term care expenditures, with the great majority of countries having essentially no private long-term care insurance market to speak of (OECD, 2011). The case of Switzerland is interesting in the sense that although private expenditure on long-term care services represents almost 1.5% of GDP, less than 0.5% of this amount is actually financed by insurance companies. In the United States, 7% of all long-term care expenses are financed by insurance (OECD, 2011). This represents between 15% and 18% of all private long-term care expenditures in the country.

The private long-term care insurance market remains small despite the important social costs associated with dependency in the last years of life. In Brown and Finkelstein (2011) we learn that less than 14% of Americans have long-term care insurance, but the reported number in Tumlinson et al. (2009) and Davidoff (2013) is closer to 10%. In Canada, Boyer et al. (2017) report even lower penetration rates.

2.3 Government programs

An important aspect of the LTC services market is the very large presence of government services which could be crowding-out private alternatives both in terms of having their own facilities and/or insurance schemes. Overall, OECD countries spend on average 1.5% of GDP on LTC services (see Figure 4 for the year 2008 and Figure 5 for the year 2014), of which only 20% can be considered as private expenditures. We can therefore conclude that, in developed countries at least, governments pay a large share of the direct costs of long-term care services. For instance, in the United States, we learn from Tumlinson et al. (2009) that Medicaid pays for approximately 70 percent of nursing home patients.

We observe in every OECD country - except Switzerland - that public expenditures in LTC services is larger than private expenditures.[5] We note, in particular, the 100% market share of public services when it comes to long-term care in Sweden, the Netherlands, Ireland, and France even though they all spend more in percentage of their GDP than the average OECD country. The good news, is that despite the demographic trend towards having older populations and the pressure it puts on health care cost at older ages, OECD countries have still been able to keep LTC expenditures (either public or private) at a relatively low levels. Nevertheless, funding of long-term care varies within a general framework of benefits that are either constant or declining in wealth.

[5] Merlis (2004) shows the wide variation of ways that OECD countries have used to finance LTC services. It goes from subsidized and mandated insurance in Germany and Japan to general tax-funded services in Sweden and Denmark, means-tested provision of services in Canada, the United Kingdom, and the United States.

This means that the perception on the insurance company side may be that there is very little demand for a product that would cover losses of at most $21,600 per year. Boyer et al. (2019) come up with a present value, at age 65, of the expected cost of long-term care services in Ontario of less than $20,000 ($13,000 in Quebec). They reach that conclusion assuming that

- half of the population aged 65 and over will require some form of nursing home;
- nursing home residents use the service for 5 years on average; and
- individuals will need a nursing home at age 80 (so 15 years later) on average.

Assuming there are fixed costs to selling LTC insurance contracts (that is, assuming a fixed insurance premium loading) of $10,000,[6] it is quite possible that many individuals' willingness-to-pay is smaller than the insurance industry's break-even premium of $30,000. If this is the case, then the low penetration of long-term care insurance can certainly be partially explained by the different levels of government in Canada offering valued and valuable LTC services, which are crowding-out the private insurance sector. This can occur even if LTC services and coverage are valued.

Brown and Finkelstein (2008) show that social insurance, and in particular Medicaid in the United States, crowds out the demand for private insurance. While acknowledging that the public provision of health services late in life can explain the lack of insurance, one can also imagine that generous retirement programs also reduce the need for long-term care insurance. They contend that high-risk individuals (i.e., those who have a high probability of having a long life) are being subsidized in the retirement and annuity market by low-risk individuals. When time comes for the high-risk individuals to purchase LTC insurance, they realize that they are richer than they should have been had their retirement not been subsidized by the low-risk individuals, and their need for LTC insurance is reduced. Low-risk individuals, seeking to separate themselves from the high-risk individuals may actually be better off not purchasing LTC insurance than subsidizing the high risk individuals a second time. The combination of generous retirement programs run by the government with adverse selection with respect to the risk of living long result in high-risk individuals wanting to be under-insured, and low-risk individuals to have little or even no insurance at all.

[6] Transaction costs which represents 33% of the total premium are not extraordinarily high.

2.4 Family and Informal help

One of the most important feature of long-term care services is the presence of what is called *informal family help*. From the National Institute on Again, we learn that most long-term care is provided at home by unpaid family members and friends.[7]

Many studies have documented the importance of informal family help; there is now a consensus (see for instance Bonsang (2009), Charles and Sevak (2005), and Van Houtven and Norton (2004)) about the substitutability between formal and informal help. A report from OECD (2011) highlights the fact that family care-takers are primarily younger women, and in particular spouses and adult daughters. Access to family support explains part of the little demand for long-term care insurance because asking help from family members is relatively easy. In addition, it may happen that there is a implied *quid pro quo* in that inheritance or bequest may be tied to the provision of informal long-term care. In addition of actively taking care of and devoting time to help elderly parents, informal help also consists in the children sharing their house or apartment with them, or moving back in with an elderly parent who is unwilling to leave his or her home (Pinquart and Sorensen, 2002).

The challenges associated with family care are multifaceted. There is the inter-generational moral hazard problem[8] in which parents, who prefer the company of their loved ones to that of the formal sector, choose to remain uninsured with respect to their potential long-term care need in order to force their children to take care of them.

A second challenge with family help is

3 The model

In our model we juxtapose the following elements: (i) the market for long-term care services in private facilities; (ii) the market for long-term care insurance; (iii) the public provision of long-term care services; and (iv) the provision of informal care by family members. To the best of our knowledge, we are the first to analyze the interaction of these four sources of long-term care services in one single analytical framework.

3.1 The market for long-term care services

We use a modification of Salop's (1979) circular city model to represent the operations of the private market for long-term care services. There are N consumers in the economy. A

[7] https://www.nia.nih.gov/health/what-long-term-care, last visited on 12 March 2019.

[8] See in particular Pauly (1990), Zweifel and Struewe (1996), Zweifel and Struewe (1997), and Courbage and Zweifel (2011).

consumer needs long-term care services with probability ρ whereas she stays healthy with probability $(1-\rho)$. Conditional on needing long-term care, we distinguish between different intensities of the required services. We denote the consumer's long-term care needs by R_m with $m \in \{1, 2, \ldots, M\}$ being a ranking of the amount of services needed. We can think of the index m as the number of activities of daily living (ADL) or instrumental activities of daily living (iADL), for which the agent needs help.[9] Therefore, we assume $R_1 < R_2 < \ldots < R_M$ and let the probability of needing help with L ADLs be given by ξ_m for $m \in \{1, 2, \ldots, M\}$.

We interpret each severity level as its own market and denote the cost of providing long-term care services in market m by c_m, with $c_1 < c_2 < \ldots < c_M$. This cost is the same for each supplier of LTC services in a particular market. For simplicity, we assume that the government can provide LTC services in a particular market at cost. Therefore, the expected cost of long-term care services per individual is $\rho \sum_{m=1}^{M} \xi_m c_m$.

We represent the market for long-term care services as a right circular cone, see Figure 6. Each severity level of long-term care needs corresponds to a horizontal cross section of the cone, yielding a circle of radius R_m for $m \in \{1, 2, \ldots, M\}$. The government is located at the vertex of the cone. The height and the slant height of the cone enclose an angle $g \in (0°, 90°)$, which represents the government's social policy in our model. The smaller g is, the greater is the vertical distance between the government and any particular long-term care market. More specifically, if v_m denotes the vertical distance between market R_m and the government, and s_m denotes the slant distance between the market R_m and the government, then we know from basic trigonometry that $\tan g = \frac{R_m}{v_m}$ and from the Pythagorean Theorem that $v_m^2 + R_m^2 = s_m^2$. Solving for s_m then yields

$$s_m = \sqrt{R_m^2 + \left(\frac{R_m}{\tan g}\right)^2} = R_m \sqrt{1 + \frac{\cos^2 g}{\sin^2 g}} = R_m \frac{1}{\sin g}, \tag{1}$$

which is a decreasing and convex function of the government's policy.[10]

Agents with long-term care needs R_m are uniformly distributed on the circumference of the corresponding long-term care market. They can either choose to obtain services from a private provider or from the government depending on their preferences and the associated

[9] Activities of daily living typically include walking, dressing, toileting (including managing incontinence), brushing teeth and eating. Instrumental activities of daily living are activities that people do as members of society such as managing finances, cooking, driving, communicating (which includes using the telephone, computer and iPads), shopping, managing medication, keeping appointments, etc. Alternatively, the American Occupational Therapy Association (`https://www.aota.org/`) identifies 12 types of instrumental activities of daily living that may be performed in a community, such as taking care of pets, observing religious holy days, or taking care of others.

[10] Indeed, we obtain

$$\frac{\mathrm{d}s_m}{\mathrm{d}g} = -\frac{R_m}{2} \frac{\cos g}{\sin^2 g} < 0 \quad \text{and} \quad \frac{\mathrm{d}^2 s_m}{\mathrm{d}g^2} = \frac{R_m}{4} \frac{1 + \cos^2 g}{\sin^3 g} > 0 \tag{2}$$

for $g \in (0°, 90°)$. The slant distance to the government decreases at a decreasing rate as g increases.

costs. Each agent in market R_m has the same distance to the government, which is the slant distance $s_m = R_m \frac{1}{\sin g}$. The agent's distance to the closest private provider of long-term care services depends on her position on the circumference relative to the position of the private providers. We assume that private providers compete against each other for profits and are therefore equidistantly distributed on the circumference. Therefore, if there are n_m providers in market R_m, they partition the circumference in n_m segments of equal length given by $2\pi R_m/n_m$. Hence, the agent's distance to her closest provider is contained in $[0, \pi R_m/n_m]$.

3.2 Demand for long-term care services

Individuals compare the costs and benefits associated with public versus private provision of long-term care services to make their decision. We assume a transportation cost of $t > 0$ to private providers of long-term. To travel to publicly provided long-term care services, a share λ of consumers incurs a transportation cost of τ (high-cost types or H-types) whereas the remainder $(1-\lambda)$ does not incur any transportation cost to the government (low-cost types or L-types). The distribution over transportation cost types is independent of the distribution over location on a particular market. We can think of the transportation cost as preference parameters that measure the perceived quality of either type of service. Everything else equal, an L-type consumer will always prefer to receive long-term care services from the government whereas an H-type consumer prefers privately provided long-term care services if $t < \tau$, given that the price is identical. Private providers in market R_m charge a price of p_m for long-term care whereas the government provides those services at cost and charges c_m. We focus on symmetric equilibria in each market so that every provider charges the same price. Later on, we will endogenize prices and the number of suppliers. The following lemma summarizes consumers' demand behavior.

Lemma 1. *A type H consumer with distance $d \in [0, \frac{\pi R_L}{n_L}]$ to her closest private provider of long-term care services will demand services from that provider if and only if*

$$td + p_m \leq \tau \frac{R_m}{\sin g} + c_m. \tag{3}$$

This condition is more likely to be satisfied the lower the transportation cost to private providers, the higher the transportation cost to the government, the closer the consumer to her next private provider, the lower the cost of private long-term care services, the higher the intensity of her long-term care needs and the smaller the policy parameter g.

Proof. Straightforward. □

Two special cases of Lemma 1 are immediate. If private providers charge a price exceeding $\tau \frac{R_m}{\sin g} + c_m$, all H-types will demand long-term care services in that particular market from the government. If, however, the price charged by private providers is below $\tau\frac{R_m}{\sin g} + c_m - t\frac{\pi R_m}{n_m}$, no H-type consumer will demand services from the government and private providers serve them. We also not that, if $p_m \geq c_m$, consumers who do not incur transportation cost to the government will also demand publicly provided long-term care services. As we will see, this is what happens in equilibrium.

4 Equilibria

We first start out with a definition what it means for the market to be in equilibrium. We will then go on to discuss what types of equilibria can arise in our model.

Definition 1 (Equilibrium)**.** A *symmetric equidistant equilibrium (SEE)* is a (I+2)-uple $(\mathbf{p}, \mathbf{n}, \mathbf{z^i})$ with $\mathbf{p}, \mathbf{n} \in \mathbb{R}_+^M$ and $\mathbf{z^i} \in [0,1]^M$ for $i \in \{1, ..., I\}$ such that in each market R_m the following holds:

(i) If n_m suppliers are equidistantly distributed and each of them charges a price of p_m, then profits are zero.

(ii) No supplier has an incentive to deviate from price p_m.

(iii) At those prices, a share z^i of type i consumers are served on the private market with the remainder being served by the government.

The underlying idea is that supply and demand must be mutually consistent for each submarket to be in equilibrium and therefore for the whole market to be in equilibrium. Suppliers charge prices such as to maximize profits but entry drives profits to zero. Also, suppliers take into account how prices affect demand and demand must be such that consumers make rational decisions between private and public long-term care services, depending on their particular preferences. In the sequel, we will construct different types of equilibria and focus on individual markets for ease of exposition.

4.1 Only one type of agents

As a benchmark case, we present the equilibrium outcome in market R_m if there is only one type of agents with transportation cost τ for government services. We define by $\delta_m = (\rho\xi_m N)/(2\pi R_m)$ a measure of how densely market R_m is populated by comparing the expected number of individuals to be served in that particular market to its geometric size. We then obtain the following result.

Proposition 1. *Market R_m is served entirely by private providers if and only if*

$$\frac{3}{2}\sqrt{\frac{tf}{\delta_m}} \leq \tau \frac{R_m}{\sin g}. \tag{4}$$

In this case, the equilibrium price for long-term care services of intensity R_m and the equilibrium number of suppliers in market R_m are given by:

$$p_m^* = c_m + \sqrt{\frac{tf}{\delta_m}} \quad \text{and} \quad n_m^* = \rho \xi_m N \sqrt{\frac{t}{f\delta_m}}. \tag{5}$$

At the other end of the spectrum, every consumer will be served by the public sector if

$$\tau \frac{R_m}{\sin g} \leq \sqrt{2}\sqrt{\frac{tf}{\delta_m}}. \tag{6}$$

In the middle (that is, for $\frac{3}{2}\sqrt{\frac{tf}{\delta_m}} \leq \tau \frac{R_m}{\sin g} \leq \sqrt{2}\sqrt{\frac{tf}{\delta_m}}$), government and private provision of long-term care services may coexist, in which case $p_m^{**} = \frac{1}{2}\tau\frac{R_m}{\sin g} + c_m$ so that there can be at most $n_m^{max} = \frac{2\pi t \sin g}{\tau}$ suppliers on this market.

We provide a proof in Appendix A.1. We point out some properties of the equilibrium prices and number of providers. Prices are positively associated with the cost of providing long-term care services in market m, the consumer's transportation cost and the fixed entry cost of suppliers. They are negatively associated with the market density; intuitively, consumers have more bargaining power in more densely populated markets, which lowers prices in equilibrium. Entry is positively associated with the consumer's transportation cost but negatively with the fixed entry cost. The effect of an increase in the market density on n_L^* depends on the specific reason why it increases. If there are more individuals in the market, the equilibrium number of suppliers rises but if the geometric size of the market decreases (i.e., if $R_m \downarrow$), then the equilibrium number of suppliers decreases. Intuitively, more individuals allow for more opportunities to provide long-term care services but a smaller geometric size has room for only a smaller number of providers.

The condition for exclusive provision of long-term care services by private providers is given in Eq. (4). Private provision of long-term care services in market R_m is more likely the lower the consumer's transportation cost to private providers, the lower the fixed entry cost for private providers, the higher the market density, the lower the government's policy parameter g and the higher the cost differential between public and private provision of long-term care services. The intensity of long-term care needs in a particular market has a two-fold effect on Eq. (4). More intense needs raise the distance of each consumer in market m from the government, making exclusively private provision of services more likely. On the other hand, more intense needs lower the market density, which makes exclusively private provision of services less likely. The net effect can be positive or negative, and the following remark sheds some light on this issue.

Remark 1. *Consider market R_m in the economy. There is a critical value R_m^{crit} with exclusively private provision of long-term care services for $R_m \geq R_m^{crit}$.*

In other words, the effect that more intense needs raise the distance of each consumer from the government dominates for high enough values of R_m and all services will be offered through the private market.

Corollary 1. *In any private market that exists (i.e., when $\tau \frac{R_m}{\sin g} \geq \sqrt{2}\sqrt{\frac{tf}{\delta_m}}$), then the active presence of government-provided services will lower prices (i.e., $p_m^{**} \leq p_m^*$), while at the same time reducing the number of suppliers of services (i.e., $n_m^{**} < n_m^*$).*

4.2 Fully separating equilibria with two type of agents

We now assume that each type is represented in the economy, that is, $\lambda \in (0,1)$. We then characterize situations where all type H consumers are served on the private market and all type L consumers are served by the government. In our notation, this corresponds to $z_m^H = 1$ and $z_m^L = 0$. Effectively, this shrinks the size of the private market, which turns out to lower the number of suppliers but increases prices relative to the case where both types receive services from private providers. Also, we need a different consistency requirement to ensure that H-type have no incentive to deviate to the government. The following proposition summarizes.

Proposition 2. *Type H consumers are served on the private market and type L consumers are served by the government if and only if*

$$\frac{3}{2}\sqrt{\frac{tf}{\lambda\delta_m}} \leq \tau \frac{R_m}{\sin g}. \tag{7}$$

In this case, the equilibrium price for long-term care services of intensity R_m and the equilibrium number of suppliers in market R_m are given by:

$$p_m^* = c_m + \sqrt{\frac{tf}{\lambda\delta_m}} \quad \textit{and} \quad n_m^* = \rho\xi_m N\sqrt{\frac{\lambda t}{f\delta_m}}. \tag{8}$$

A proof is provided in Appendix A.4. The condition in (7) is saying intuitively that τ needs to be large enough in order for H-types to prefer services from the private market whereas type L consumers always prefer services from the government because they do not mind traveling to the government and services are provided at cost. Recall that λ denotes the proportion of high types in the economy. We see that the equilibrium price is negatively associated with λ while the number of suppliers is positively associated with λ. As a result, the condition on τ is less restrictive the more H-types are in the economy. The reason is that, if more H-types are served in the private market, prices for privately provided long-term care services decrease, which makes the private sector relatively more attractive compared to the government.

4.3 Exclusively public provision

We finally turn to the case where long-term care services are exclusively provided by the government. Intuitively, if the travel cost to the government is small enough, consumers will always prefer to obtain services there because the government can offer them at cost whereas private providers would always charge above marginal-cost prices. We will show two results in this section. First, we identify the condition under which no provider can ever hope to make profits on the private market so that all services will be provided by the government. Second, we will see that this condition leaves a "gap" in the parameter space, which suggests the question of what happens for intermediate parameter values. We will show that no equilibrium exists in that case because the market price induced by competitive pressure between providers and the government fails to be individually-optimal for each specific provider.

Proposition 3. *All consumers will demand services from the government if*

$$\tau \frac{R_m}{\sin g} \leq \sqrt{2}\sqrt{\frac{tf}{\lambda \delta_m}}. \tag{9}$$

Intuitively, if τ is low enough, agents do not mind traveling to the government and private providers cannot hope to make a profit on the market. Indeed, if they tried and condition (9) holds, the resulting profit would be negative. By comparing conditions (7) and (9), we see that there are intermediate values for τ that are not covered by our existing analysis because $\sqrt{2} < 3/2$. This raises the question whether this gives rise to intermediate equilibria with some H-type consumers being served on the private market and the remainder going to the government for services. The following results shows that the answer to this question is negative.

Remark 2. *With a continuous number of firms, a situation with $z_m^L = 0$ and $z_m^H \in (0,1)$ cannot be an equilibrium.*

The intuitive reason is that the price that would make a specific allocation between private and public provision of long-term care services consistent is not compatible with profit-maximizing behavior of suppliers. Said differently, any sharing of H-type consumers between the government and the private market is such that, at the margin, suppliers would always prefer to charge marginally higher or lower prices to increase profits. We conclude from our analysis thus far that equilibria have a so-called "bang-bang" structure in the sense that $z^L = 0$ and $z^H \in \{0, 1\}$.

5 Insuring privately provided services

In a next step, we discuss how insurance affects market outcomes. For simplicity, we assume a coinsurance contract that reimburses a portion of the cost of long-term care services. We use Davidoff (2013) to argue that coinsurance provisions are the reasonable way to examine the problem as he writes:

> Most US long-term care policies offer reimbursement for expenditures incurred associated with nursing homes, some home care services, (and) limited reimbursement for informal care provided by family and friends (p. 1042).

We denote by $\alpha \in [0, 1]$ the coinsurance rate, which specifies the percentage of cost that is reimbursed by the insurer. Therefore, the consumer retains a fraction $(1 - \alpha)$ of the cost, which affects her trade off when comparing private and public provision of long-term care services. As in the previous section, we compare different types of equilibria and explain how they are affected by insurance.

5.1 No government involvement

As in the previous section, we first revisit the case of no government involvement, assuming a measure λ of agents are of type H. The argument in this case follows closely the argument in Nell et al. (2009), which is embedded in our case with $\lambda = 1$. The fact that consumers do not pay the full price for long-term care services because a portion is covered by insurance distorts the competition between suppliers, which leads to excess entry and higher prices. This makes government involvement relatively more attractive. The following proposition summarizes.

Proposition 4. *For coinsurance rate α, market R_m is served entirely by private providers if and only if*

$$\frac{3}{2}\sqrt{\frac{tf}{\lambda\delta_m(1-\alpha)}} \leq \tau\frac{R_m}{\sin g}. \tag{10}$$

In this case, the equilibrium price for long-term care services of intensity R_m and the equilibrium number of suppliers in market R_m are given by:

$$p_m^* = c_m + \sqrt{\frac{tf}{\lambda\delta_m(1-\alpha)}} \quad \textit{and} \quad n_m^* = \rho\xi_m N\sqrt{\frac{t}{f\lambda\delta_m(1-\alpha)}}. \tag{11}$$

The proof is obtained by combining the arguments in Appendix A.1 with those in Nell et al. (2009). The comparative statics discussed previously are still valid. In addition, we note that insurance raises prices and the number of suppliers in the market, which makes private provision of long-term care services less attractive. Said differently, to the extent that privately provided services are insurable, condition (10) is less likely to be satisfied and the government needs to be "less close" in order for consumers to reconsider their choice and leave the private market. This can be attributed to the price distortion effects that insurance has on privately provided long-term care services.

5.2 Exclusively public provision

We now turn to the last case, which is that all consumers receive services from the government. Given the effects of insurance on the private market, this case will become increasingly more likely to the extent that insurance plays a role on the private market. More formally, we state the following result.

Proposition 5. *If a coinsurance rate of α applies to privately provided services, all consumers in market R_m will demand services from the government if*

$$\tau \frac{R_m}{\sin g} \leq \sqrt{2}\sqrt{\frac{tf(1-\alpha)}{\lambda \delta_m}} - \alpha c_m. \tag{12}$$

Note that condition (12) nests condition (9) in the absence of insurance for $\alpha = 0$. Agents who know that, for a given market they will go to the government, are most likely not going to purchase any insurance so that $\alpha = 0$ makes sense here so that we are back on condition (9).

5.3 Partial government involvement

When governments are partially involved, somehow, with private insurance still possible, then we have a market for insurance that will depend on the agents' willingness to travel to the government to obtain services.

Proposition 6. *Knowing that market R_m is served entirely by private providers if and only if equation 10 holds and that every consumer will be served by the public sector if and only if equation 12 (or* (9)*) holds, then it must be that government and private providers share the market when*

$$\frac{3}{2}\sqrt{\frac{tf}{\lambda \delta_m (1-\alpha)}} \geq \tau \frac{R_m}{\sin g} \geq \sqrt{2}\sqrt{\frac{tf(1-\alpha)}{\lambda \delta_m}} - \alpha c_m. \tag{13}$$

Assume that all consumers are served by the government. Then a single private provider charging a price of p will attract consumers within a distance $d = \frac{1}{t}\left(\tau \frac{R_m}{\sin g} + c_m - (1-\alpha)p\right)$.

This long-term care provider, which is considering the possibility of entering this market, therefore solves the following problem

$$\max_{p\in[c_m,\tau\frac{R_m}{\sin g}+c_m]} \Pi(p)=\left\{\lambda\frac{\rho\xi_m N}{2\pi R_m}\frac{2}{t}\left(\tau\frac{R_m}{\sin g}+c_m-(1-\alpha)p\right)(p-c_m)-f\right\},$$

which yields a profit-maximizing price of $p^{\#}=\frac{1}{2(1-\alpha)}\left[\tau\frac{R_m}{\sin g}+(2-\alpha)c_m\right]$. At this price, the supplier will attract consumers to the left and to the right of its location within a distance of $d^{\#}=\frac{1}{t}\left(\tau\frac{R_m}{\sin g}+c_m-p^{\#}\right)=\frac{1}{2t}\left(\frac{\tau R_m}{\sin g}+\alpha c\right)$. Consequently, there can be at most $n_m^{\#}=\frac{2\pi R_m}{2d^{\#}}=t\frac{2\pi R_m}{\left(\frac{\tau R_m}{\sin g}+\alpha c\right)}$ suppliers on this market.[11]

It is interesting to note that the presence of insurance reduces the number of service providers that compete on this market. The opposite was true when government was not involved at all, as we can see from equation 11.

At this price, the profit[12] for any provider of services is given by

$$\Pi(p^{\#})=\lambda\delta_m\frac{1}{2t}\left(\tau\frac{R_m}{\sin g}+\alpha c_m\right)^2\left(\frac{1}{1-\alpha}\right)-f. \tag{14}$$

If condition (9) holds, then Equation (14) is non-positive so that not even one provider enters the market.

5.4 Fully separating equilibria

Now we turn to a situation where H-type consumers are served on the private market and L-type consumers are served by the government ($z_m^H=1$ and $z_m^L=0$). Again, this has implications for the equilibrium number of suppliers and prices on the market. The following proposition summarizes.

Proposition 7. *Assume a coinsurance rate of α. Then, type H consumers are served on the private market and type L consumers are served by the government if and only if*

$$\frac{3}{2}\sqrt{\frac{tf}{\lambda\delta_m(1-\alpha)}}\leq\tau\frac{R_m}{\sin g}. \tag{15}$$

---

[11] Note that we fall back to $p_m^{**}=\frac{1}{2}\tau\frac{R_m}{\sin g}+c_m$, $d^{**}=\frac{1}{2t}\left(\tau\frac{R_m}{\sin g}\right)$, and $n_m^{**}=\frac{2\pi t\sin g}{\tau}$ when $\alpha=0$.

[12] Again, we note that if we set $\alpha=0$, then we fall back to the same condition as when there was no insurance.

In this case, the equilibrium price for long-term care services of intensity R_m and the equilibrium number of suppliers in market R_m are given by:

$$p_m^* = c_m + \sqrt{\frac{tf}{\lambda\delta_m(1-\alpha)}} \quad \textit{and} \quad n_m^* = \rho\xi_m N\sqrt{\frac{\lambda t}{f\delta_m(1-\alpha)}}. \tag{16}$$

The proof is obtained by combining the arguments in Nell et al. (2009) with those in Appendix A.4. We point out two particularities of our model. The fact that L types are served by the government anyway shrinks the size of market R_m relative to the case where everybody would obtain services from private providers. As it turns out, the effects of insurance interact with the size of the market. Specifically, insurance has a larger effect on prices on a smaller compared to a larger market and p_m^* increases faster as a function of insurance coverage when the market is small than when it is large. For the number of suppliers the opposite is the case. Insurance stimulates excess entry into the market for long-term care services but this effect is mitigated by the smaller size of the market. Overall though, the threshold value on τ has the same property as the price of services, which is that the smaller market exacerbates the effects of insurance. As a result, a small λ coupled with a high α make it increasingly harder for condition (15) to be satisfied.

6 Policy and welfare

A bunch of graphs

7 Discussion and conclusion

Davidoff (2013) writes

> Private insurers face an environment in which public insurance, family care, and home equity provide substitutes for a large fraction of the population. There is reason to suspect that households who demand private insurance despite the presence of substitutes may be bad actuarial risks (p. 1051).

We present in this paper a self-contained model in which the interactions between all the players play a very important role. It may be that Pauly (1990) was right in that individuals are rationally not purchasing long-term care insurance. We provide in this paper theoretical reasons for this to occur, which include the presence of government services and the locally monopolistic competition among suppliers of services, let alone insurance suppliers.

We show many other things

"It is a far, far better thing that I do, than I have ever done; it is a far, far better rest that I go to than I have ever known."

Charles Dickens

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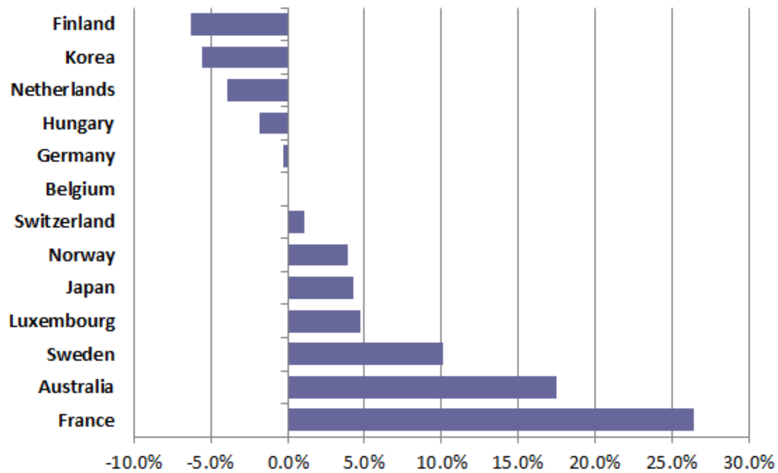

Figure 1: The change in the share of individuals aged over 65 years using institutional care in selected OECD countries between 2003 and 2007. The values are calculated as the average share from 2006 and 2008 divided by the average share from 2002 and 2004, minus 1.

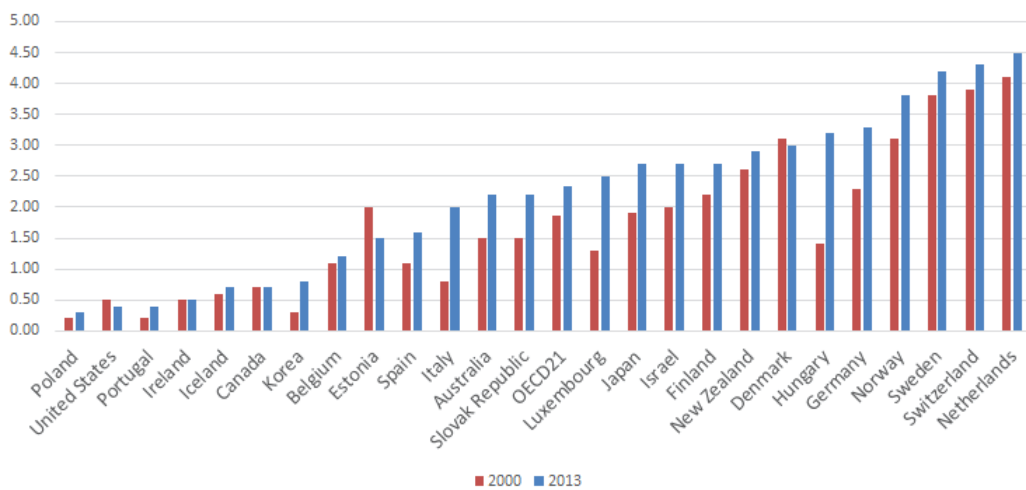

Figure 2: Proportion of the total population receiving long-term care services in selected OECD countries in 2000 and 2013 (source: OECD, 2015; Health at a Glance http://dx.doi.org/10.1787/health-data-en).

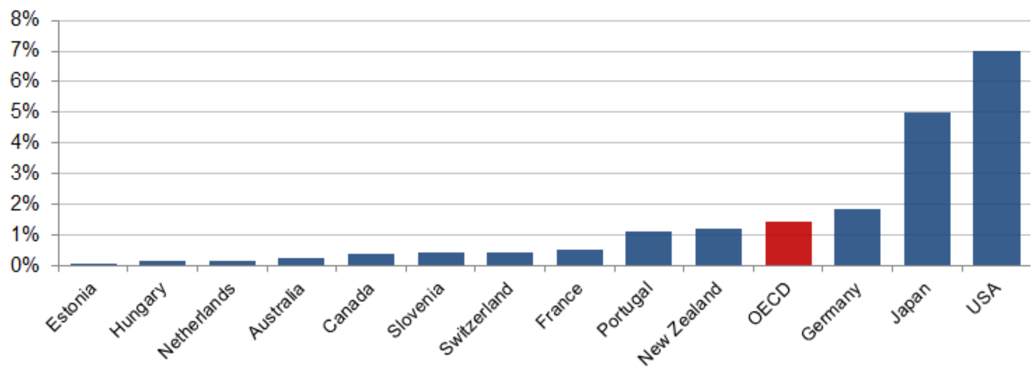

Figure 3: Private long-term care insurance market as a percentage of total long-term care expenditures for selected OECD countries in 2008 (source: OECD, 2011; http://www.oecd.org/els/health-systems/47887332.pdf).

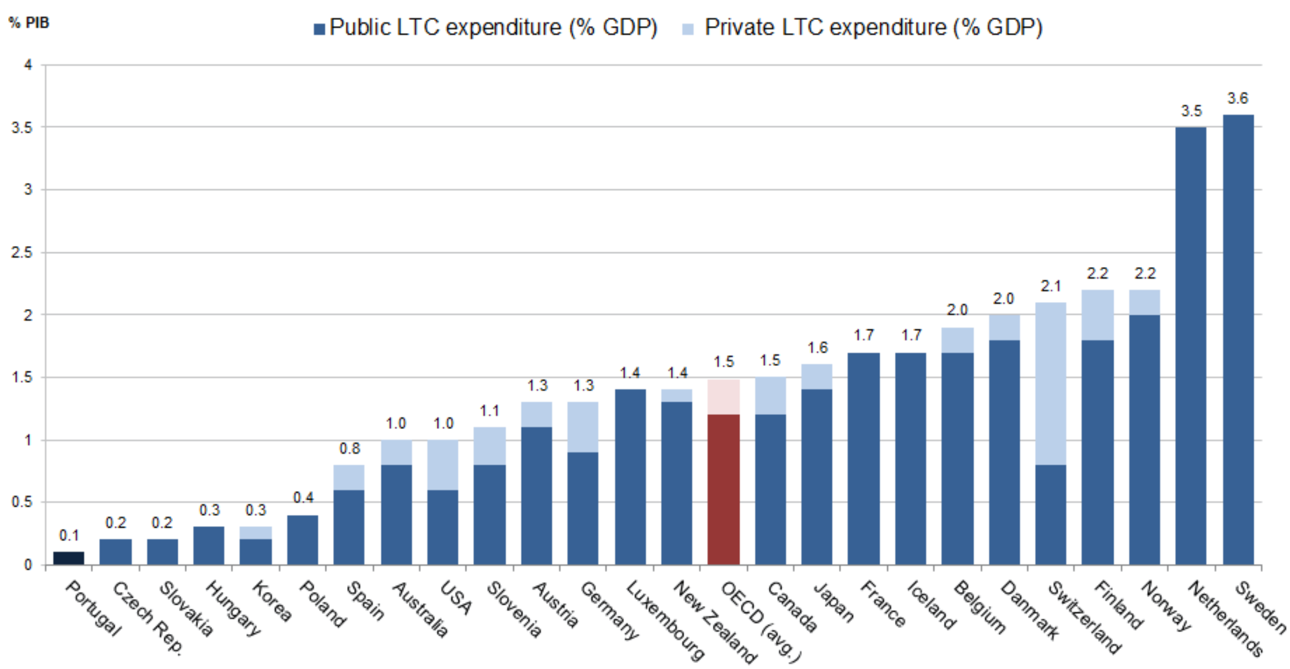

Figure 4: Public and private long-term care expenditures as a percentage of gross domestic product for 30 OECD countries in 2008 (source: OECD, 2011; http://www.oecd.org/els/health-systems/47887332.pdf).

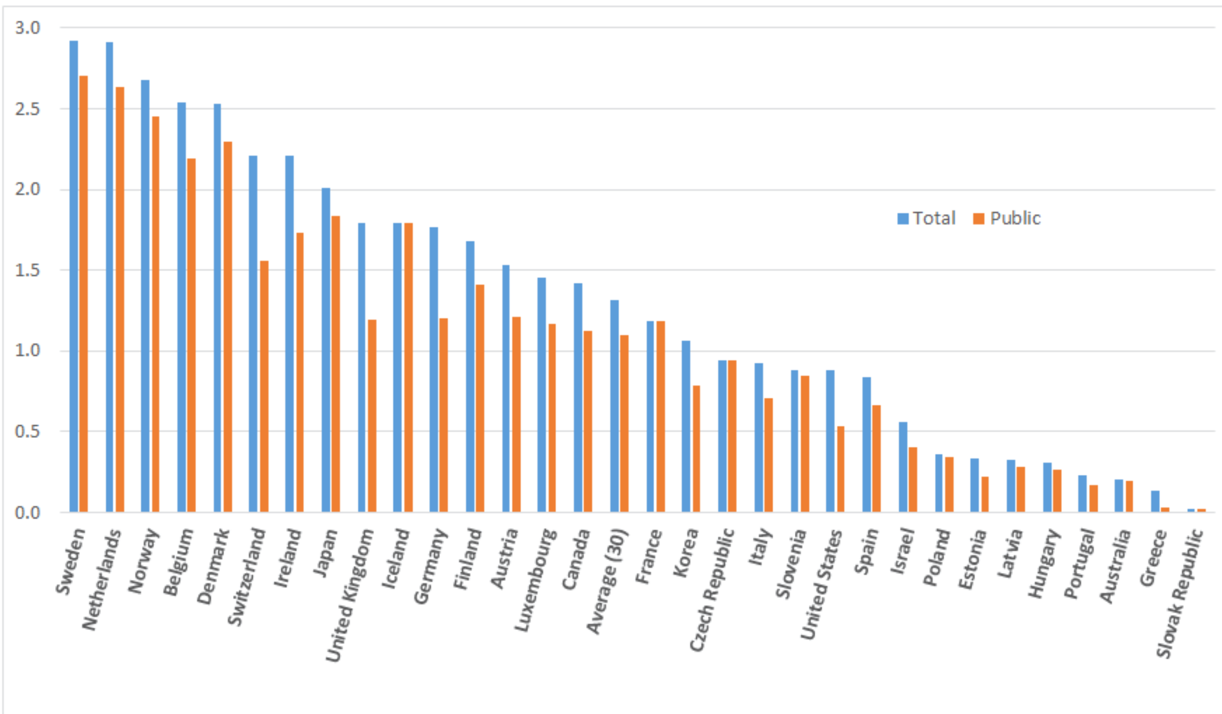

Figure 5: Total and public sector spending on long-term care services as a percentage of the country's GDP for the year 2014. Source: OECD Health Statistics (2017)

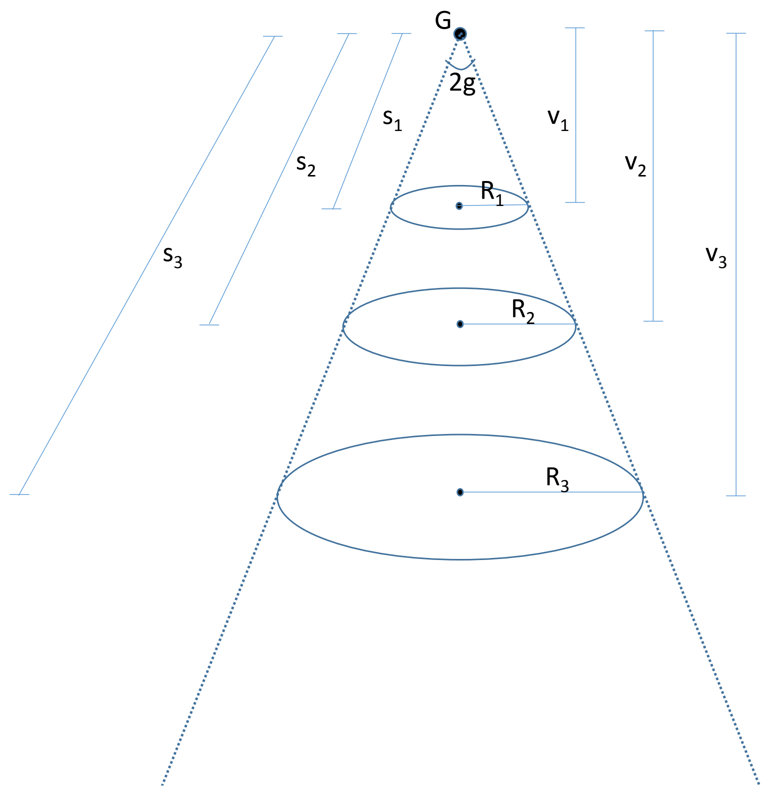

Figure 6: View from the side of the long-term care service diagram of our economy. The severity level determines the amount of care needed and is given by R_m, with $m \in \{1, 2, \ldots, M\}$. The provision of services by the government is located at the vertex of the cone. The vertical distance between the government and each long-term care market is given by v_m, the slant distance between the government and each long-term care market is given by s_m.

A Mathematical proofs

A.1 Proof of Proposition 1

The proof of Proposition 1 has three parts.

Part 1. There are $\rho\xi_m N$ individuals in market m who need services. These $\rho\xi_m N$ people are uniformly distributed on the market's circumference of length $2\pi R_m$. Assume that there are n_m private providers in market m and that they are equidistantly located on the circle. The marginal consumer between two suppliers i and j is located at a distance d from supplier j such that

$$p^j + td = p^i + t\left(\frac{2\pi R_m}{n_m} - d\right), \tag{17}$$

where p^i and p^j are the prices of suppliers i and j, respectively. Therefore, each equidistant supplier attracts a share of

$$\frac{2d}{2\pi R_m} = \frac{p^i - p^j}{2\pi R_m t} + \frac{1}{n_m} \tag{18}$$

individuals in market m. Taking the per-capita surplus of $(p^j - c_m)$ and the fixed cost f for entry into account yields the following profit for supplier j:

$$\Pi(p^j) = \rho\xi_m N\left(\frac{p^i - p^j}{2\pi R_m t} + \frac{1}{n_m}\right)(p^j - c_m) - f. \tag{19}$$

Maximizing over p^j and using symmetry implies $p^j = p^i = c_m + \frac{2\pi R_m t}{n_m}$. We insert this into supplier j's profit function and set it equal to zero. This renders n_m^* in Eq. (5). Inserting n_m^* into p^j then renders p_L^* in Eq. (5).

Part 2. For consistency, we need to verify that no consumer has an incentive to demand services from the government instead. Per Lemma 1, this is the case if and only if

$$t\frac{\pi R_m}{n_m^*} + p_m^* \leq \tau^L \frac{R_m}{\sin g} + c_m. \tag{20}$$

Inserting p_L^* and n_L^* from Eq. (16) and rearranging then yields condition (4).

Part 3. For the last part of the proof, assume that all consumers are served by the government. A single private provider that charges a price of p will attract consumers within a distance $d = \frac{1}{t}\left(\tau\frac{R_m}{\sin g} + c_m - p\right)$. This is positive as long as $p \leq \tau\frac{R_m}{\sin g} + c_m$. The provider therefore solves

$$\max_{p\in[c_m,\tau\frac{R_m}{\sin g}+c_m]} \Pi(p) = \left\{\frac{\rho\xi_m N}{2\pi R_m}\frac{2}{t}\left(\tau\frac{R_m}{\sin g} + c_m - p\right)(p - c_m) - f\right\},$$

which yields a profit-maximizing price of $p^{**} = \frac{1}{2}\tau\frac{R_m}{\sin g} + c_m$. At this price, the supplier will attract consumers to the left and to the right of its location within a distance of $d^{**} = \frac{1}{t}\left(\tau\frac{R_m}{\sin g} + c_m - p^{**}\right) = \frac{\tau}{2t}\left(\frac{R_m}{\sin g}\right)$. There can be at most $n_m^{**} = \frac{2\pi R_m}{2d^{**}} = \frac{2\pi R_m}{\frac{\tau}{t}\left(\frac{R_m}{\sin g}\right)} = \frac{2\pi t \sin g}{\tau}$ suppliers on this market.

At this price, the profit for any provider of services is given by

$$\Pi(p^*) = \frac{\delta_m}{2t}\left(\tau\frac{R_m}{\sin g}\right)^2 - f. \tag{21}$$

If condition (6) holds, then Equation (21) is non-positive so that not even one provider enters the market.

A.2 Proof of Remark 1

Define $g(R_m) = \tau\frac{R_m}{\sin g} - \frac{3}{2}\sqrt{\frac{tf}{\delta_m}}$; then, condition (4) is equivalent to $g(R_m) \geq 0$. We obtain $\lim_{R_m \to 0} g(R_m) = 0$ and

$$g''(R_m) = \frac{3}{8}\frac{1}{R_m^2}\sqrt{\frac{tf}{\delta_m}} > 0$$

so that $g(R_m)$ is convex. Furthermore, $g(R_m^{crit}) = 0$ for $R_m^{crit} = \frac{9\pi \sin g t f \tau}{2\rho\xi_m N}$, which completes the proof because $g(R_m) < 0$ for $R_m \in (0, R_m^{crit})$ and $g(R_m) > 0$ for $R_m > R_m^{crit}$.

A.3 Proof of Corollary 1

Assume that $\tau\frac{R_m}{\sin g} \geq \sqrt{2}\sqrt{\frac{tf}{\delta_m}}$, so that a private market exists.

The active presence of government-provided services will lower prices (i.e., $p_m^{**} \leq p_m^*$) provided that $\tau\frac{R_m}{\sin g} \leq 2\sqrt{\frac{tf}{\delta_m}}$. This is always the case from condition (4).

The active presence of government-provided services will reduce the number of suppliers of services. In other words, the maximum number of suppliers, $n_m^{**} = \frac{2\pi t \sin g}{\tau}$, is always smaller than n_m^* if and only if $\tau\frac{R_m}{\sin g} \geq \sqrt{\frac{ft}{\delta_m}}$. This is always true since a private market exists only when the more restrictive condition (6) holds.

A.4 Proof of Proposition 2

Similar arguments as in Appendix A.1 demonstrate that

$$p_m^* = c_m + \sqrt{\frac{tf}{\lambda\delta_m}} \quad \text{and} \quad n_m^* = \rho\xi_m N\sqrt{\frac{\lambda t}{f\delta_m}},$$

by multiplying $\rho\xi_m N$ with λ and solving accordingly. Consistency requires for H-type consumers to prefer privately provided services over those provided by the government, which is the case if and only if

$$t\frac{\pi R_m}{n_m^*} + p_m^* \leq \tau^H\frac{R_m}{\sin g} + c_m.$$

Inserting p_m^* and n_m^* and rearranging then yields the condition on τ^H. Furthermore, consistency requires for L-type consumers to prefer services provided by the government over privately provided services. Per Lemma 1, this is the case if and only if

$$p_m^* \geq \tau^L \frac{R_m}{\sin g} + c_m,$$

which then provides the condition involving τ^L.

A.5 Proof of Proposition 3

Assume that all consumers are served by the government and that a single private provider is trying to attract H-types. If it charges a price of p, it will attract consumers within a distance $d = \frac{1}{t}\left(\tau \frac{R_m}{\sin g} + c_m - p\right)$, which is positive as long as $p \leq \tau \frac{R_m}{\sin g} + c_m$. The provider therefore solves

$$\max_{p \in [c_m, \tau \frac{R_m}{\sin g} + c_m]} \left\{ \Pi(p) \frac{\rho \xi_m N}{2\pi R_m} \lambda \frac{2}{t} \left(\tau \frac{R_m}{\sin g} + c_m - p\right)(p - c_m) - f \right\},$$

which yields a profit-maximizing price of $p^* = \frac{1}{2}\tau \frac{R_m}{\sin g} + c_m$. At this price, profit is given by

$$\Pi(p^*) = \frac{\delta_m \lambda}{2t} \left(\tau \frac{R_m}{\sin g}\right)^2 - f,$$

which is non-positive if and only if condition (9) holds.

A.6 Proof of Remark 2

Suppose there is a share $z \in (0, 1)$ of H-types in market R_m who are served by the private market with the remainder being served by the government. Further assume that there are n_m equidistanly distributed suppliers on the circumference who charge a price of p_m for services. Suppliers share aggregate profits in the particular market such that each supplier's profit is given by

$$\Pi = \frac{z \lambda \rho \xi_m N}{n_m}(p_m - c) - f.$$

The suppliers partition the circumference into n_m segments of length $\frac{2\pi R_m}{n_m}$, and each of those segments loses its middle portion to the government. To be consistent with a share of z being served by the private market, each segment is subdivided into two pieces of length $\frac{2\pi R_m z}{2n_m}$ close to private providers and a middle piece of length $\frac{2\pi R_m(1-z)}{n_m}$ of consumers who go to the government. As a result, the marginal consumer is located at a distance of $\frac{2\pi R_m z}{2n_m}$ to her closest private provider. For indifference to hold, we obtain

$$p_m + \frac{2\pi R_m z t}{2n_m} = \tau \frac{R_m}{\sin g} + c_m.$$

Substituting the implied price into the supplier's profit function, we arrive at

$$\Pi = \frac{z \lambda \rho \xi_m N}{n_m} \left(\tau \frac{R_m}{\sin g} - \frac{2\pi R_m z t}{2n_m}\right) - f$$

Competition implies $\Pi = 0$, which yields the following quadratic equation for the number of suppliers:

$$2fn_m^2 - 2z\lambda(\rho\xi_m N)\tau\frac{R_m}{\sin g}n_m + z^2\lambda(\rho\xi_m N)(2\pi R_m)t = 0.$$

Its discriminant is given by

$$D = 4(\lambda z)^2(\rho\xi_m N)^2\left[\left(\tau\frac{R_m}{\sin g}\right)^2 - 2\frac{ft}{\lambda\delta_m}\right].$$

Introducing the abbreviations $A = \tau\frac{R_m}{\sin g}$ and $B = \sqrt{\frac{ft}{\lambda\delta_m}}$, we see that $D > 0$ if and only if $A^2/B^2 > 2$. This is consistent with condition (9) because if $A \leq \sqrt{2}B$, all H-type consumers are served by the government (i.e., $z = 0$). We also know from condition (7) in Proposition 2 that $A^2/B^2 < 9/4$ because otherwise all H-type consumers are served by the private market (i.e., $z = 1$).

Solving the quadratic equation yields the following two possible solutions for the number of suppliers

$$n_m^* = \frac{z\lambda(\rho\xi_m N)}{2f}\left(A \pm \sqrt{A^2 - 2B^2}\right),$$

both of which are positive. We then substituting those solutions into the condition on the price and obtain

$$p_m^* = c_m + A - \frac{B^2}{A \pm \sqrt{A^2 - 2B^2}} = c_m + A - \frac{1}{2}\left(A^2 \mp \sqrt{A^2 - 2B^2}\right),$$

which is independent of z. For market R_m to be in equilibrium, suppliers cannot have an incentive to deviate from this price. Looking at a particular supplier's situation, a marginal increase or decrease of the price by an amount ε would have the following effect on profit:

$$\Pi(\varepsilon) = \left[\frac{z\lambda\rho\xi_m N}{n_m^*} - \frac{2\varepsilon}{2\pi R_m t}\right](p_m^* + \varepsilon - c_m) - f.$$

A marginally higher price loses some costumers to the government but increases the per-capita surplus while a marginally higher price attracts some costumers from the government but decreases the per-capita surplus. By construction, $\Pi(0) = 0$, and for the supplier not wanting to deviate, we need $\Pi'(0) = 0$ as well. This last condition rearranges to

$$2\pi R_m tf = (A^2 - B^2) \pm \sqrt{A^2 - 2B^2},$$

which does not depend on z and represents a complex restriction on the exogenous parameters. If this restriction is not satisfied, suppliers always have an incentive to deviate from the price that would make any sharing of H-type consumers between private and public provision of long-term care services consistent and equilibrium fails to exist. In case the restriction is satisfied, any sharing is consistent and we can simply assume z to be negligibly small and all H-type consumers to be served by the government.