Competition, Asymmetric Information, and the Annuity Puzzle: Evidence from a Government-run Exchange in Chile

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Abstract

Purchasing an annuity insures an individual against the risk of outliving their money, by promising a steady stream of income until death. Despite the fact that retirement asset allocation models predict high annuitization rates, private annuity markets tend to function poorly, with low annuitization rates and high markups. Chile provides a counterexample to this phenomenon, as over 60% of eligible retirees purchase annuities from the private market and observed markups are low. This paper shows that Chilean social security policy promotes private annuitization, in contrast to US social security policy. We do so by building a lifecycle consumption-savings model and showing through calibrations that the Chilean setting is likely to have lower levels of adverse selection and is more robust to market unraveling than the US. We then use a novel administrative dataset on all annuity offers made to Chilean retirees between 2004 and 2013 to estimate a flexible demand system built on top of the consumption-savings model. The model estimates allow us to simulate how the Chilean equilibrium would shift under alternative regulatory regimes. We find that reforming the Chilean system to more closely resemble the US Social Security system, by introducing mandatory annuitization of a fraction of wealth at the actuarially fair rate, raises prices and can lead to full market unravelling. When comparing welfare between both systems, however, we find that retirees with high valuations for annuitization tend to prefer a US style system while retirees with low valuations tend to prefer the Chilean system. The former tend to value the

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high annuity rate from Social Security, even if further annuitization is not possible, while the latter dislike mandatory annuitization. Highly heterogeneous preferences across retirees appear to drive annuity market behavior.

1 Introduction

Income during retirement is essential to the financial stability of older populations, and increasing life expectancies have increased the difficulty of providing this stability. Typically, retirees receive income from a combination of government-provided social insurance programs and private retirement income products, such as life annuities. Annuities offer a fixed or minimally varying stream of payments for the remainder of the annuitant's life span, as well as optional features such as guaranteed payments to the annuitant's heirs or the delay of payments until a later age. In the United States, most households choose not to purchase annuities with their retirement savings, despite having relatively low levels of retirement income from other sources. This phenomenon, often called the *annuitization puzzle*, has spurred a large economic literature attempting to explain retiree behavior. The literature has proposed that adverse selection has contributed to the low equilibrium rate of annuitization in the United States.

Chile provides an important counterexample to the US experience - more than 60% of eligible retirees voluntarily buy private annuities. This paper investigates the role of regulation in paving the way for a successful private annuity market, using novel administrative data from Chile. Specifically, the paper asks whether changing the regulatory structure of Chile's annuity market to make it more similar to the United States' greatly increases adverse selection and leads to low equilibrium annuitization. To do so, we first solve an optimal consumption-savings problem for multiple consumer types, and show in calibrations that with the same underlying primitives one can find full annuitization in the Chilean system and market unraveling in the US. We then introduce a demand system that allows us to nonparametrically estimate the distribution of these types, which allows us to revisit the previous calibration analysis with empirically founded distributions of unobserved heterogeneity. Armed with these estimates, we can also simulate other policy reforms, as well as compare welfare for different consumer groups across different retirement systems.

In 2004, Chile instituted an innovative government-run exchange that all retirees must use to access their savings. The exchange is a virtual platform which transmits consumer information and preferences to all annuity sellers (life insurance companies), solicits offers from any company willing to sell to that consumer, and organizes the offers by generosity to facilitate the retiree's decision process. Retirees may also choose not to purchase an annuity and instead to draw down the balance of their retirement savings account, according to a schedule set by the government. This alternative is called "programmed withdrawal". Programmed withdrawal allows retirees to leave more wealth for their heirs if they die early, and provides more liquidity early in retirement. Therefore, it is more valuable as a vehicle for bequests and liquidity, rather than as a source of insurance against excessive longevity. The government's role is primarily in transmitting information between firms and consumers through the exchange, without limiting price discrimination or constraining consumer and firm choice.

Using novel data on every annuity and programmed withdrawal offer provided on this platform from 2004 to 2012, we document three striking facts about the Chilean annuity market. First, more than 60% of single retirees voluntarily purchase annuities. Second, the prices they pay are low, with the average accepted annuity being 3% less generous than an actuarially fair annuity. And third, we show that the Chilean market exhibits adverse selection - individuals who annuitize are longer lived, conditional on observables, than those who do not. That is, we find that the regulatory regime in Chile supports a functioning annuity market despite the presence of the typical culprit for market unraveling.

To tease out the drivers of these facts, we calibrate a life cycle model and calculate implied annuity demand and average cost curves. In Chile, the market exhibits a relatively inelastic demand function. In contrast, introducing rules akin to those of the United States leads to more elastic annuity demand, meaning average cost increases faster than willingness to pay for annuities. As a result, the addition of any administrative cost or market power may cause the US market to unravel. Interestingly, the calibration implies that local approximations to the supply and demand elasticities, estimated around the observed equilibria, would lead to misleading conclusions. That is, the local elasticity of demand is relatively low in both Chile and the US, which would imply that if supply was perfectly competitive and provided with zero administrative costs, both Chile and the US would see nearly full annuitization.

These facts suggest that Chile's regulatory regime combats adverse selection, relative to the counterfactual of US-style social security and insurance regulations. The calibration further implies that to quantify the welfare effects of adverse selection, we must identify the distribution of preferences for financial instruments that is underlying this market. Linear approximations and other reduced form methods will be inaccurate, given the highly nonlinear shape of the demand and average cost curves. We proceed to estimate a novel structural model of annuity demand that allows us to nonparametrically identify the distribution of private information in the market. The model, based on Fox et al. (2011), proceeds in two steps. First, we solve the optimal consumption-savings problem for every annuity and programmed withdrawal offer conditional on a retiree type. From this solution we obtain the value of each contract given a retiree type. We then use the estimator presented in Fox et al. (2011) and Fox et al. (2016) to non-parametrically estimate the distribution of types that rationalizes observed choices.

Demand estimates show significant unobserved heterogeneity among retirees. Specifically, we find that women have higher bequest motives than men, and that individuals with higher wealth outside the system are also longer lived. Richer individuals and more short lived individuals are both less risk averse, but not very significantly. Higher bequest motives are also correlated with lower risk aversion. Furthermore, using the estimated distribution of unobserved types we find that reforming the Chilean system to make it more similar to the US setting results in significantly worse annuity prices and can lead to full market unraveling.

Welfare comparisons between the Chilean and the US regime show that no system strictly dominates the other - there are types who prefer the Chilean equilibrium and there are types who prefer the US equilibrium. The main driver behind this heterogeneity is underlying heterogeneity in types, which creates different valuations for annuitization. Individuals with low valuations for annuitization tend to value the Chilean system above the US system, as it gives them the ability to place all their money into programmed withdrawal. This increases payouts in the first years after retirement, and it also allows for more money to be left to heirs, which some types value significantly. On the other hand, individuals with high valuations for annuitization tend to value the US system above the Chilean system, even if the annuity market unravels, as Social Security provides a more generous annuity payout and covers a large fraction of wealth. This suggests that policy interventions that allow retirees to exchange their Social Security payments for assets with better liquidity and inheritance properties, like programmed withdrawal, can be a way to increase retiree welfare.

This work brings together two strands of the literature - one investigating the annuity puzzle and the other modeling and estimating equilibrium in markets with asymmetric information. The annuity puzzle literature focuses on explaining the low level of annuitization in the US. Mitchell et al. (1999) and Davidoff et al. (2005) document the high utility values of annuities, and show they are robust to a variety of modeling assumptions. Friedman and Warshawsky (1990) document the relatively high price of annuities in the market, relative to other investments, which can partially explain the annuity puzzle. Lockwood (2012) demonstrates how significant bequest motives further lower the value of an annuity.

Scholarship on markets with asymmetric information have focused on detecting adverse selection, and using structural econometrics to model private information. Chiappori and Salanie (2000) and Finkelstein and Poterba (2014) test for asymmetric information in the reduced form. Einav et al. (2010) use a structural model to estimate demand for annuities in mandatory UK market, and study the interaction between adverse selection and regulatory mandates. Like Einav et al. (2010), we ue a structural model to back out the distribution of retirees' private information. Our contribution is to nonparametrically identify private information without making any assumptions on firm pricing behavior. We also solve for counterfactual equilibria when significant changes are made to the pension system, accounting for equilibrium effects.

The rest of the paper is structured as follows: section 2 introduces the main features of the Chilean retirement exchange; section 3 presents descriptive evidence on the functioning of this system; section 4 develops the lifecycle model of consumption and savings used for both calibrations and demand estimation; section 5 uses a calibration to show why differences in regulation between Chile and the US can lead to differences in annuity market equilibria even with the same demand and supply primitives; section 6 presents our demand estimation framework, provides details on the empirical implementation, and discusses identification; section 7 estimates the distribution of underlying types and uses demand estimates to simulate counterfactual annuity market equilibria and welfare under different regulatory regimes; and section 8 concludes.

2 The Chilean Retirement Exchange

Chile has a privatized pension savings system. Individuals who are employed in the formal sector must contribute 10% of their income to a private retirement savings account administered by a Pension Fund Administrator (PFA). In order to access the accumulated balance upon retirement, retirees must utilize a government-run exchange called “SCOMP”[1]. The exchange can be accessed either through an intermediary, such as an insurance sales agent or financial advisor, or directly by the individual at a pension fund administrator. Individuals can enter SCOMP at any time after they have accumulated more wealth than the legal minimum. In practice, since the minimum wealth requirement falls significantly after certain age thresholds (60 for women and 65 for men), most retirees enter then exchange at that point or after. Individuals provide the exchange with their demographic information, private savings account balance, and the types of annuity contracts they want to elicit offers for (choices ae deferral of payments, purchase of a guarantee period to provide payouts after death, and fraction of total wealth to annuitize[2]).

There are between 13 and 15 firms participating in this exchange between 2004 and 2013. Once an individual enters the exchange, each firm simultaneously receives the following information: gender, age, marital status, age of the spouse, number and age of legal beneficiaries other than the spouse, pension account balance, and the set of contracts for which offers have been requested.[3] Armed with this information, each firm sets individual and contract type specific offers. They can also not bid on some or all of the contracts the individual requested. There is no regulation impeding price discrimination based on any of the characteristics firms observe through SCOMP.

Retirees can opt not to accept an annuity offer, but instead to take an alternative product, called programmed withdrawal (“PW”). Programmed withdrawal provides a front-loaded drawdown of pension account funds according to a regulated schedule, with two key provisions. First, whenever the retiree dies, the remaining balance in their savings account is given to their heirs. Second, if the retiree is sufficiently poor and lives long enough for payments to fall below a minimum pension, the government will top up PW payments to reach this minimum level.[4] When an individual chooses the PW option, their retirement balance remains at a PFA, which invests it in a low risk fund. As a result, PW payments are stochastic, although the variance is small. Figure 1 below shows one realization of PW payments for a female who retires at 60, and compares it to the average annuity offer that individual received.

Retirees receive all annuity offers and information about PW on an informational document provided by SCOMP. The document begins with a description of programmed withdrawal and a sample drawdown path (figure 2). Then, annuity offers are listed, ranked within contract type first by generosity (figure 3). Along with the life insurance company’s name, retirees are also informed of their risk rating. This information is relevant, as retirees are only partially insured against the life insurance company going bankrupt.[5] After receiving this document, retirees can accept an offer or enter a bargaining stage. Retirees can physically

[1] Sistema de Consultas y Ofertas de Montos de Pensión.

[2] With significant restrictions. In our sample, fewer than 10% of retirees were eligible to not annuitize their total savings

[3] Other beneficiaries include children under 18 (25 if they are attending college), former spouses

[4] The threshold for receiving the minimum pension is being below the 60% percentile of total wealth according to the “Puntaje de Focalización Previsional”

[5] To be precise, the government fully reinsures the MPG plus 75% of the difference between the annuity payment and the MPG, up to a cap of 45 UFs. A UF is an inflation-indexed unit of account used in Chile. In December 12, 2017, a UF was worth 40.85 USD. In practice, there has been only one bankruptcy since the private retirement system’s introduction in the 1980s, and that company’s annuitants received their full annuity payments for 124 months after bankruptcy was declared. Only after that period did their payments fall to the governmental guarantee. For more details on the bankruptcy process, see (in Spanish) `http://www.economiaynegocios.cl/noticias/noticias.asp?id=35722`

Figure 1: A simulated path of payments made under PW to a retiree who retires at 60, compared to the average annuity that retiree is offered

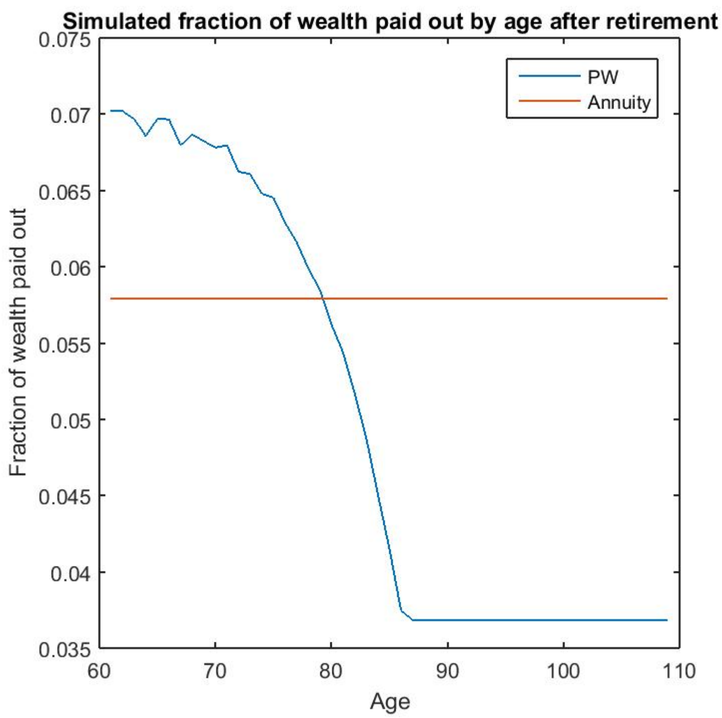
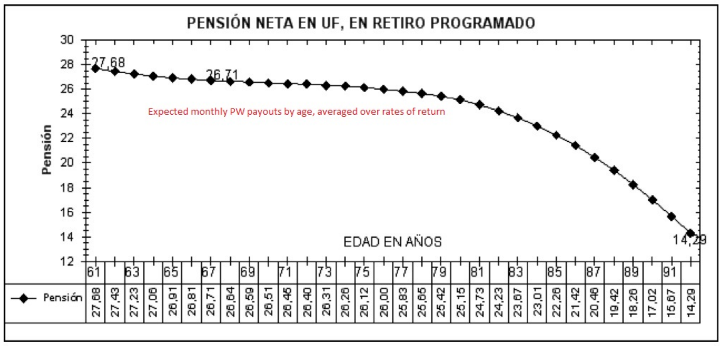


Figure 2: Sample printout of programmed withdrawal information conveyed to retiree



Monto Pensión mensual promedio: 24,18 UF

Monto Comisión mensual promedio: 0,30 UF

Figure 3: Sample printout of annuity offers for one contract type

MODALIDAD RENTA VITALICIA INMEDIATA

RENTA VITALICIA INMEDIATA SIMPLE Annuitize full wealth, 0 guarantee, 0 deferral

N° Oferta	Compañía de Seguros de Vida (Brand Name)	Pensión final Mensual sin Retiro de Excedente UF	Pensión final Mensual en UF Considerando un retiro de excedente de 0,00 UF	Pensión con retiro de Excedente Máximo		Clasificación de riesgo de la Compañía de Seguros (2)
				Pensión final Mensual UF	Excedente UF	
43872093	CRUZ DEL SUR	26,61	<- Monthly payment		Risk rating ->	AA-
43872099	RENTA NACIONAL	26,58				BBB-
43872083	METLIFE	26,52				AA
43872100	CORPSEGUROS	26,34				AA-
43872094	PRINCIPAL	26,28				AA
43872097	CORPVIDA	26,26				AA-
43872084	EUROAMERICA VIDA	26,25				AA-
43872090	PENTA VIDA	26,25				AA-
43872091	OHIO NATIONAL	26,24				AA
43872098	SURA	26,21				AA
43872095	CN LIFE	25,90				AA
43872092	BICE VIDA	25,86				AA+
43872085	CHILENA CONSOLIDADA	25,59				AA
43872086	CONSORCIO VIDA	25,36				AA+

travel to any subset of firms that gave them offers through SCOMP to bargain for a better price, for some or all of the contracts they are interested in.[6] On average, these outside offers represent a modest increase in generosity over offers received within SCOMP, on the order of 2%. Finally, the individual can choose either to buy an annuity from the final choice set or to take PW. Individuals that don't have enough retirement wealth to fund an annuity above a minimum threshold amount per month will receive no offers from firms, and must take PW.

Our primary source of data is the individual-level administrative dataset from SCOMP from 2004 to 2013, which includes the retiree's date of birth, gender, geographic location, wealth, and beneficiaries. This data includes contract-level information about prices, contract characteristics and firm identifiers. We observe the contract each retiree chooses, including if they choose not to annuitize, and can compare the characteristics of the chosen contract to the other choices they had, including offers received during the bargaining stage. Overall, we observe 230,000 retirees and around 30 million annuity offers. We supplement

[6]Firms are not allowed to lower their offers in this stage

Table 1: Average characteristics of our sample and of accepted annuity contracts

	N	Mean	10th Pctile	Median	90th Pctile
Panel A: Retiree Characteristics					
Total wealth (UFs)	45091	2378.488	937.42	1958.91	4190.91
Female (dummy)	45091	0.759	0	1	1
Age	45091	62.566	60	62	66
Married	45091	0.207	0	0	1
Insurance agent	45091	0.284	0	0	1
Died in two years	45091	0.016	0	0	0
Choose annuity	45091	0.688	0	1	1
Panel B: Contract Characteristics					
Choose dominated offer	31062	0.194	0	0	1
Monthly payment (UFs)	31062	11.132	5.08	9.26	19.06
Deferral years	31062	0.532	0	0	2
Guarantee months	31062	124.873	0	120	216
Money's worth ratio	31062	1.00288	0.9445	1.0050	1.0740

this data with two external datasets. First, we include external data about the life insurance companies making offers, such as their risk rating[7]. Second, we merge individual-level death records obtained from the Registro Civil in mid 2015.

Throughout our sample period, annuity contracts for married males are regulated to be joint life annuities, while this is only the case for married females after 2007. Furthermore, for retirees with children younger than 18, life insurance companies must continue paying out a fraction of the annuity payment upon the retirees' death until the child turns 18.[8] For simplicity, we will focus our analysis on the subsample of retirees with no beneficiaries. This subsample purchases only single life annuities that insure their own longevity risk. Mortality risk can therefore be included directly, avoiding more complex actuarial calculations involving the annuitant's beneficiaries and their mortality probabilities. As a result, our dataset consists of 53,356 individuals who receive annuity offers and accept either programmed withdrawal or an annuity within our sample period.

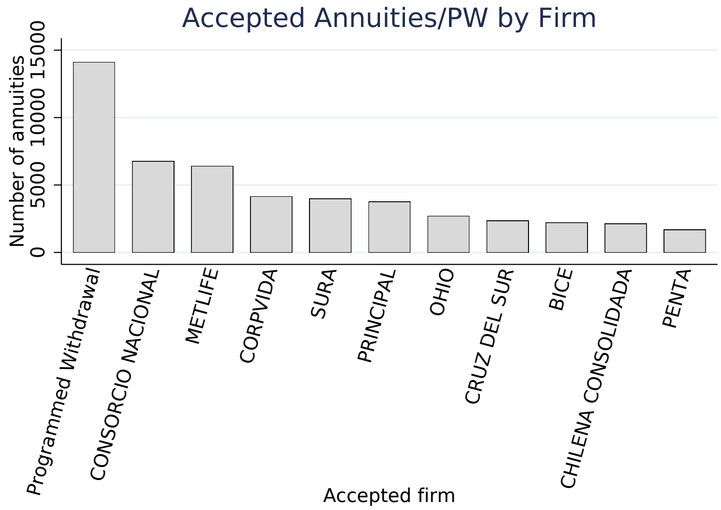
Table 1 presents summary statistics for this sample. Panel A reports statistics for all individuals, while Panel B reports statistics of accepted annuity offers. The annuitization rate for this sample is 68.8%, and the probability of death by two years after retirement is 1.6%. Note that a disproportionate fraction of our sample is female, as all married women without eligible children who retired on or before 2007 were offered single life annuities. Roughly 28% of the individuals in the sample entered their information into SCOMP through an agent of a life insurance company, and 19.4% of accepted annuity offers are dominated.[9] There is significant heterogeneity across accepted guarantee periods and deferral periods, with most individuals accepting immediate annuities and a median guarantee period of 10 years.

[7]While this variable is presented to retirees, it was not made available to us directly in the SCOMP dataset.

[8]Children younger than 25 are also covered if they are college, until either they graduate or turn 25.

[9]We call an offer dominated if there is another offer of the same contract type with weakly better risk rating and a higher payout amount

Figure 4: Contracts accepted by subsample of retirees, where the leftmost bar represents programmed withdrawal and the others refer to annuities sold by the top 10 annuity providers



3 Descriptive Evidence

There are three striking facts from the annuities market in Chile that emerge from descriptive analysis. First, the fraction of individuals voluntarily choosing annuities is high - nearly 70% of retirees choose an annuity. Second, the market for annuities is fairly unconcentrated, with each of the top ten firms getting a significant share of annuitants (figure (4)). And third, retirees receive annuity offers that are marked up on average only by 5% over the actuarially fair annuity calculated using the distribution of mortality observed in the data, although there is significant heterogeneity in the population over these markups.

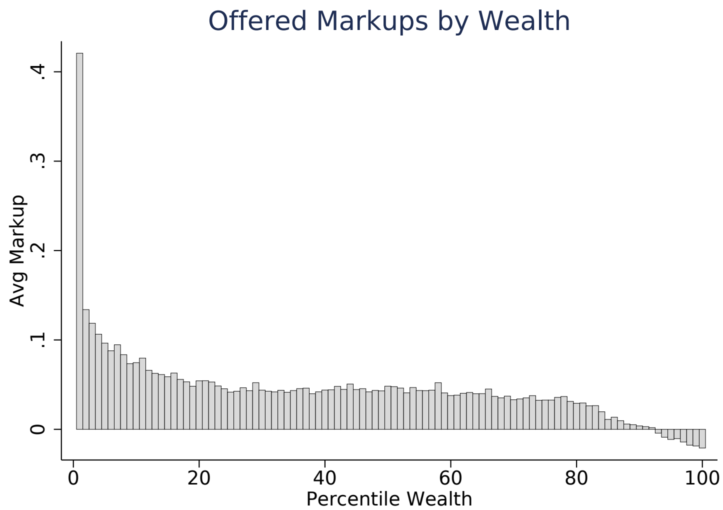
Calculating Annuity Value

To identify this, we calculate actuarially fair annuities by modelling the hazard rate of death ($h_j(t)$) as a Gompertz distribution with a different scale parameter for each demographic type j (bins of age, gender, municipality, and wealth level). The shape parameter γ is fixed and the scale parameter is modeled as $\lambda_j = e^{x_j\beta}$. The resulting hazard rate is given by:

$$h_j(t) = \lambda_j e^{\gamma t} \tag{1}$$

Since we observe death before 2015, we can estimate this model directly for our sample. Using the results of this estimation, we can predict expected mortality probabilities for each individual and calculate the net present cost of an annuity with a monthly payout z_t, discounted at rate r. The predicted survival probabilities (dependent on age, gender, wealth, and municipality) are denoted as $\{\hat{\pi}_{it}\}$. The NPV of an annuity can then

Figure 5: Markup over actuarially fair price by wealth percentile, where .1 corresponds to a net present value of the annuity being .9*wealth



be calculated as:

$$NPV(z_i) = \sum_{t=0}^{T} \frac{\hat{\pi}_{it} z_i}{(1+r)^t} \tag{2}$$

Naturally, the value of the annuity payout depends on the total retirement savings the retiree gives the life insurance company (denoted by w_i). We calculate percentage markup over cost (equal to the inverse of the moneys worth ratio minus one) as:

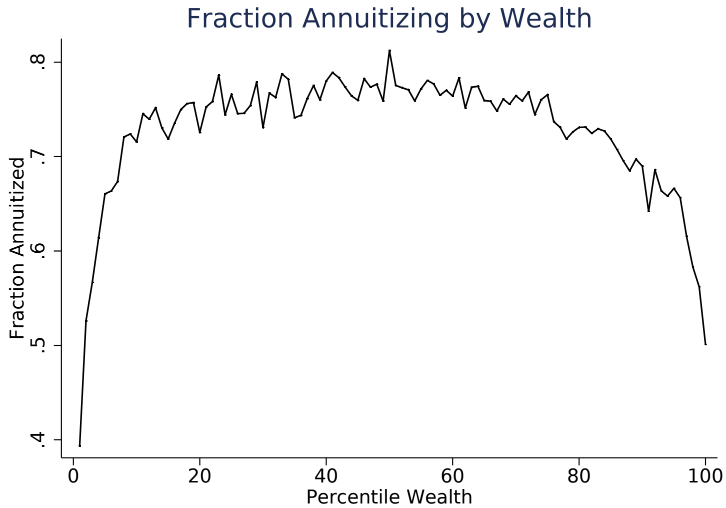
$$m_i = \frac{w_i - NPV(z_t)}{NPV(z_t)} = \frac{1}{MWR_i} - 1 \tag{3}$$

Figure 5 shows the average markup over the actuarially fair annuity that retirees are offered, by their wealth percentile, for our no-beneficiary subsample. To construct the discount rate, we calculate the yield curve for every month in our dataset using data on UF-denominated Chilean Central Bank Bonds.[10]

Pricing shows evidence of price discrimination, with the lowest wealth retirees getting prices that are as high or higher than the US average of 0.1-0.15. The highest wealth retirees, on the other hand, are offered actuarially fair, or better, annuities. Figure 6 shows the probability of choosing an annuity differentially by wealth percentile. The probability of taking an annuity is low for the lowest wealth and highest wealth individuals. The first finding is consistent with the pricing evidence: low wealth individuals receive worse offers, and as a result are more likely to select into programmed withdrawal. Note that because life insurance companies cannot make offers below the minimum pension, these low wealth individuals are receiving

[10]One, two, five, ten, twenty and thirty year bonds. One, two, and thirty year bonds are not available for the entirety of our sample period.

Figure 6: Fraction of retirees choosing annuities over PW by percentile of wealth



offers that are slightly above this threshold. Servicing these annuities is more costly, as firms anticipate coordinating with the government to transfer the top-up amount to the annuitant. As a result, fewer firms bid on low wealth annuitants and bids are worse.[11]

A first pass at comparing the value of annuities relative to PW is to repeat the exercise done in prior literature, which solves for the amount of annuitized (or PW-funding) wealth that provides equal utility to 1 unit of non-annuitized wealth. This amount is the money's worth ratio ("MWR"). Mitchell et al. (1999) perform this calculation for US retirees without a bequest motive facing actuarially fair annuities, and find an MWR of 0.7. Results from MWR calculations in our setting are shown in table 2. We find somewhat similar results for actuarially fair annuities in Chile - if the retiree has no bequest motives, she would be willing to give up 21.1% of her wealth to get an actuarially fair annuity. With a bequest motive of 2.5%[12], the MWR is 0.90, so an annuity is worth giving up 10.4% the value of non-annuitized wealth. In both cases, it is clear that annuities are valuable products. The analogous calculation for PW is enlightening - relative to no annuitization, a retiree without a bequest motive has a PW MWR of 0.925, meaning they are willing to pay 7.5% of their wealth to obtain access to PW. With a bequest motive, getting access to PW is worth 4.5% of her wealth. It should not be surprising that PW has an MWR below one, as the minimum pension guarantee provides some annuitization value. The main takeaway from this exercise, then, is to show that PW is providing relatively high net value. Retirees choosing annuitization in Chile, therefore, are likely not doing so because PW is a bad product.

[11]It is also important to note than an offer that is better than actuarially fair does not imply that the firm is losing money on the transaction - one would not expect funds to be invested wholly in UF-denominated Chilean Central Bank bonds, but rather on riskier assets with greater expected returns.

[12]40 dollars of bequeathed wealth are equivalent to 1 of own wealth.

Table 2: Money's worth ratio of programmed withdrawal and annuities

	Annuity	PW
No Bequest	0.789	0.925
Bequest = 2.5%	0.896	0.955

Selection and Market Power

Though the market is functioning remarkably well, many features of the data reflect standard intuitions about annuity markets worldwide. First, there is significant adverse selection into annuity purchase. To demonstrate this, we run the standard positive correlation test, introduced by Chiappori and Salanie (2000). In our implementation of this test, we regress the probability the retiree dies within two years of retirement, regressed on a dummy for annuity choice. Table 3 shows this baseline correlation in column 1. Columns 2 and 3 check the robustness of the result after controlling for observable characteristics of the individual and the requests the individual makes for annuity offers. This is the full set of information life insurance companies receive about retirees. We use the estimating equation:

$$d_{death,i} = \gamma_{annuitize} d_{annuitize,i} + X'\Pi_D$$

Covariates X should include all characteristics can be priced on by firms. The purpose of including controls is to make sure that selection is on unobservable characteristics - selection on observable characteristics can be reflected by a change in price, while selection on unobservables cannot be. A negative correlation means that retirees buying annuities are less likely to die than retirees that choose programmed withdrawal. Results show that annuitants are significantly more long lived than those choosing programmed withdrawal, even conditional on characteristics that firms can price on. To put this number in context, we run the correlation test using a Gompertz hazard model (reported in figure 12 in Appendix A). From this, we can estimate the relative life expectancies of annuitants, separately from non-annuitants. For a modal population of female retirees in 2010, retiring without help from an intermediary, we estimate that annuitants live on average 7 months longer than non-annuitants.

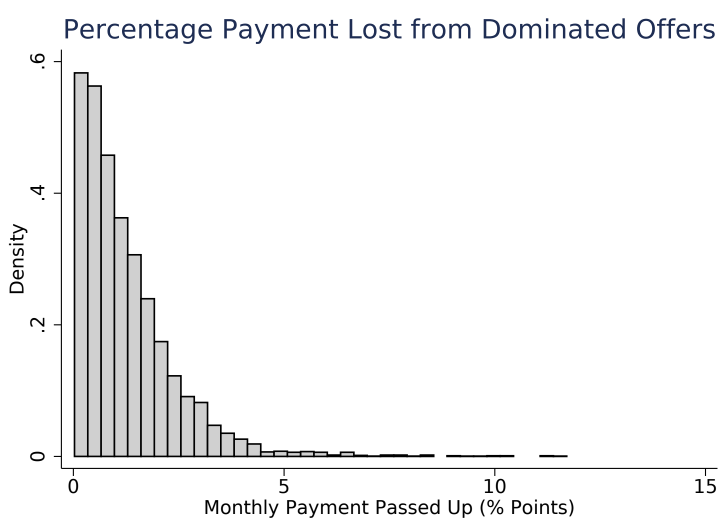
In addition to adverse selection, one could also worry about market power and non-financial preferences for firms, particularly considering that 19.4% of annuitants accept a dominated offer. Figure 7 describes the percentage of the accepted offer that the retiree could have gained from taking the dominating offer. The average loss is on the order of 1.3% of the accepted offer, a relatively small amount.

Therefore, despite the standard concerns regarding adverse selection and market power, Chile's regulatory regime supports the existence of a healthy voluntary annuity market. Our goal is to estimate the primitives governing demand for these products, and to then simulate how reforming the Chilean regulatory framework to make it more similar to the US shifts the market equilibrium. The following section introduces the model we will use to value each contract.

Table 3: Chiappori & Salanie positive correlation test

	(1) Death by 2 yrs	(2) Death by 2 yrs	(3) Death by 2 yrs
Choose annuity	-0.00740**	-0.00399**	-0.00434**
	(0.00137)	(0.00137)	(0.00156)
Individual characteristics		✓	✓
Request characteristics			✓
Observations	45091	45091	45091
Base group mean		0.015	
		(0.121)	

Figure 7: Percentage of payments foregone when accepting a dominated offer



4 Model

This section develops a model to value annuity and programmed withdrawal offers given a vector of individual characteristics and unobserved preferences. In section 6, we will embed the model into a discrete choice demand system to obtain estimates of unobserved preferences, and use these estimates to evaluate counterfactuals where we change the regulatory environment. The primary contribution of this model is to account for multiple dimensions of retiree private information, allowing preferences to be far more heterogeneous than in prior work. Specifically, the model allows for private information about mortality risk, bequest motives, risk aversion, and total wealth.

Since individuals are making choices over financial instruments that differentially shift money over time, change exposure to longevity risk, and vary the assets that are bequeathed upon death, a suitable model needs to capture these salient features. In particular, we use a finite-horizon consumption-savings model with mortality risk and the potential for utility derived from inheritors' consumption. Consider the problem of a particular individual who faces a set of annuity and programmed withdrawal offers. To obtain the value of each offer, the individual needs to solve for the optimal state-contingent consumption path, taking into account uncertainty about their own lifespan, about the probability that each life insurance firm will go bankrupt and about the return accrued by programmed withdrawal investments.

Before introducing the individual's optimization problem, some additional notation is needed. Fix an individual and firm, so we can suppress those subscripts. Let $t = 0$ denote the moment in time when the individual retires, and let T denote the terminal period in our finite horizon problem. Let ω denote outside wealth (the amount of assets held outside the pension system), γ denote risk aversion, and δ denote the discount factor. Let $d_t = \{0,1\}$ denote whether the individual is alive (0) or dead (1) in period t, and $\{\mu_\tau\}_{\tau=1}^{T}$ denote the vector of mortality probabilities [13]. Following Carroll (2011), let c_t denote consumption in period t, m_t the level of resources available for consumption in t, a_t the remaining assets after t ends, and b_{t+1} the "bank balance" in $t+1$.

For the purposes of specifying the optimal consumption-savings problem given an annuity offer, we also need to define q_t, which denotes whether the firm is bankrupt (1) or not (0) in period t, and the vector of bankruptcy probabilities for the offering firm $\{\psi_{j,\tau}\}_{\tau=1}^{T}$ [14]. With these objects, we can write the annuity payment in period t conditional on d_t, q_t, the deferral period D and the guarantee period G as $z_t(d_t, q_t, D, G)$.

With this notation, and suppressing individual and firm subscripts, we can write the individual's optimal

[13] Clearly, $d_0 = 0$ and $\mu_0 = 0$

[14] Naturally $q_0 = 0$ and $\psi_0 = 0$

consumption problem given an annuity offer as:

$$\max E_0 \left[\sum_{\tau=0}^{T} \delta^{\tau} u(c_\tau, d_\tau) \right] \tag{4}$$

$$s.t.$$

$$a_t = m_t - c_t \, \forall t$$

$$b_{t+1} = a_t \cdot R \forall t$$

$$m_{t+1} = b_{t+1} + z_{t+1}(d_{t+1}, q_{t+1}, D, G) \, \forall t$$

$$a_t \geq 0 \forall t$$

Where $R = 1 + r$, and r is the real interest rate, which we assume is deterministic and fixed over time.[15] Note that we are imposing a no borrowing constraint: there can be no negative end of period asset holdings. This assumption greatly simplifies the problem from a computational perspective, and to the best of our knowledge Chilean financial institutions do not allow individuals to borrow against future annuity or PW payments.[16]

The exogenous variables evolve as follows:

$$d_{t+1} = \begin{cases} 0 \text{ with probability } (1-\mu_{t+1}) \text{ if } d_t = 0 \\ 1 \text{ with probability } \mu_{t+1} \text{ if } d_t = 0 \\ 1 \text{ if } d_t = 1 \end{cases} \tag{5}$$

$$q_{t+1} = \begin{cases} 0 \text{ with probability } (1-\psi_{t+1}) \text{ if } q_t = 0 \\ 1 \text{ with probability } \psi_{t+1} \text{ if } q_t = 0 \\ 1 \text{ if } q_t = 1 \end{cases} \tag{6}$$

$$z_t(d_t, q_t, D, G) = \begin{cases} z \text{ if } q_t = 0 \text{ and } ((d_t = 0 \text{ and } t \geq D) \text{ or } (d_t = 1 \text{ and } D \leq t < G + D)) \\ \rho(z,t) \cdot z \text{ if } q_t = 1 \text{ and } ((d_t = 0 \text{ and } t \geq D) \text{ or } (d_t = 1 \text{ and } D \leq t < G + D)) \\ 0 \text{ otherwise} \end{cases} \tag{7}$$

$$m_0 = \omega, d_0 = 0, q_0 = 0 \tag{8}$$

[15]Annuity and PW offers in Chile are expressed in UFs, an inflation-adjusted currency, so everything in the model is in real terms.

[16]It is also increasingly difficult to keep a checking account, credit cards, and home loans open as individuals age. For example, see (in Spanish) http://www.emol.com/noticias/Nacional/2018/07/04/912082/Pinera-anuncia-que-terminara-con-discriminacion-por-edad-en-servicios-bancarios-que-afecta-a-adultos-mayores.html

Where $\rho(z,t)$ is the annuity payment when the firm goes bankrupt:

$$\rho(z,t) = \begin{cases} MPG_t & \text{if } z \leq MPG_t \\ MPG_t + \min((z - MPG) * 0.75, 45) & \text{if } z > MPG \end{cases} \tag{9}$$

and MPG is the minimum pension guarantee. For the purposes of this model, we will assume that the MPG is fixed over time. Assume that the utility derived from consumption when alive is given by the following CRRA utility function:

$$u(c_t, d_t = 0) = \frac{c_t^{1-\gamma}}{1-\gamma} \tag{10}$$

whereas if the individual dies at the beginning of period t, her terminal utility at t is given by evaluating the CRRA at the expected value of remaining wealth:

$$u(d_t = 1) = \beta \cdot \frac{\left(m_t + E[\sum_{\tau=t+1}^{G} \delta^{t-\tau} z_\tau(1, q_\tau, D, G)]\right)^{1-\gamma}}{1-\gamma} \tag{11}$$

and is equal to zero thereafter.[17]

To obtain the value of an annuity offer, which is the present discounted value of the expected utility of the optimal state-contingent consumption path, we solve this problem by backward induction. At the terminal period, the problem is simple and has an analytic solution, but for periods earlier than T it must be solved numerically. We use the Endogenous Gridpoint Method (EGM) (Carroll (2006)) to solve this problem, obtaining $V^A(0,0;\pi)$, the present discounted value of the expected utility of consumption obtained from following the optimal state-contingent policy path given an annuity offer and the vector π of parameters[18]. See Appendix C for the full derivation of the Euler equations and the computational details of the numerical solution.

Valuing a programmed withdrawal (PW) offer requires solving a related, but slightly different, problem. In this setting there is no deferral or guarantee period, or bankruptcy risk for the asset. Furthermore, inheritors automatically receive all remaining balances as a bequest upon death. All of these factors simplify the problem relative to the annuity problem. However, a significant complication arises: PW payouts are a function of the amount of money left in the PW account, which varies stochastically with market returns. As a result, the PW stock in period t, PW_t, becomes an additional state variable. Taking these differences into account, the individual's PW optimization problem, which gives us the value of accepting a PW offer

[17] This assumption implies that individuals are not risk averse about the remaining uncertainty after death. If they were, we would need to calculate expected utility instead of the utility of the expectation. From a practical perspective, this is unlikely to matter much, as the only case where remaining wealth is stochastic is for annuity offers with a guarantee period from firms who have not gone bankrupt, as wealth left to inheritors in this case is still subject to bankruptcy risk. Since bankruptcy risk is small, and most deaths will occur after the guarantee period expires, we are comfortable making this assumption.

[18] Outside wealth ω, bequest motive β, mortality probabilities $\{\mu\}_{t=1}^{T}$, risk aversion γ, and bankruptcy probabilities $\{\psi\}_{t=1}^{T}$

from firm a, is:

$$\max E_0\left[\sum_{\tau=0}^{T}\delta^{\tau}u(c_t,d_t)\right] \tag{12}$$
$$s.t.$$
$$a_t = m_t - c_t\,\forall t$$
$$b_{t+1} = a_t\cdot R_{t+1}\,\forall t$$
$$m_{t+1} = b_{t+1} + z_{t+1}(PW_{t+1},d_{t+1},f)\,\forall t$$
$$a_t \geq 0\,\forall t$$

where $z_t(PW_t,d_t,f)$ denotes the programmed withdrawal payout in period t conditional on pension balance PW_t, death status, and f, the commission rate charged by the firm. The death state and initial conditions are as before (Equation 5), and the remaining exogenous variables evolve as follows:

$$z_t(PW_t,d_t,a) = \begin{cases} max[z_t(PW_t)\cdot(1-\tau_a), MPG] \text{ if } d_t = 0 \\ 0 \text{ if } d_t = 1 \end{cases} \tag{13}$$

$$PW_{t+1} = (PW_t - z_t(PW_t))\cdot R_t^{PW} \tag{14}$$

The PW payout function $z_t(PW_t)$ is described in detail in Appendix B. All PFAs are governed by the same PW function, and conditional on the PW balance, will pay out the same amount up before the commission f. As a result, if PFAs provided the same returns over time, the amount of money that is withdrawn every year from the PW account would be the same across PFAs, and only how that money is distributed between the retiree and the PFA would vary across companies. We will assume that in fact PFAs provide the same returns on PW investments, as this simplifies the problem and is not far from reality, where PFA returns vary slightly for the safe investment portfolios where PW balances are invested [19]. Let R_t^{PW} be the return to programmed withdrawal investments. We assume $\ln R_t^{PW} \sim N(\ln(R+r) - \sigma^2_{PW}/2, \sigma^2_{PW})$, with r denoting the equity premium of PW over the market interest rate R. Finally, MPG is the minimum pension guarantee. Every individual who takes PW is guaranteed a payout of at least MPG, and the difference between $z_t(PW_t)$ and MPG (when $z_t(PW_t) < MPG$) is funded by the government. Finally, utility derived from consumption is as before, while upon death utility is:

$$u(d_t = 1) = \beta\cdot\frac{(m_t + PW_t)^{1-\gamma}}{1-\gamma} \tag{15}$$

As for annuities, we solve this problem numerically by backwards induction using EGM, and obtain $V^{PW}(0, PW_0; \pi)$, the present discounted value of the expected utility of consumption obtained from following the optimal

[19] Illanes (2017) documents this in detail

state-contingent policy path given an initial PW balance of PW_0 and the vector π of parameters. See Appendix C for the full derivation of the Euler equations and the computational details of the numerical solution.

5 Calibration

In this section, we calibrate the previous life cycle model and calculate the value of an annuity relative to programmed withdrawal for different mortality beliefs. With a given distribution of mortality expectations, we map these utilities to a model of market equilibrium by calculating demand for annuities and average cost of supplying annuities. We then change the alternative to annuitization, following Mitchell et al. (1999), to mimic US-style social security, and study how the market equilibrium changes.

We model heterogeneity in mortality risk as shifts over the mortality tables used by the Chilean pension authorities[20]. More precisely, given a retiree's age, these tables give us a mortality probability vector. We introduce heterogeneity as shifts in the individuals' age, so that a 65 year old retiree with a x year mortality shifter has the mortality probability vector of a $65+x$ year old. This allows us to introduce unobserved heterogeneity in mortality risk in a parsimonious way, at the cost of assuming that all shifts in mortality preserve the shape of the regulatory agencies' tables[21].

For ease of exposition, all other parameters in the model are fixed in this section. The representative retiree is drawn from the data - a 60 year old female, retiring in 2007 with relatively high wealth. Parameters of the utility function are taken from previous literature when possible. The risk aversion parameter is 3, interest rate is 3.18% (yearly), the standard deviation of the mortality shifter is 7, the bequest motive parameter is 10, and the fraction of the retiree's total wealth annuitized is 20%[22]. The utility obtained from the retiree annuitizing is calculated based on the choice of an simple annuity with no deferral or guarantee period for the full pension balance. The alternative to annuitization is full allocation of funds to programmed withdrawal, which follows the standardized schedule set by the government. That is, in this section we are abstracting away from heterogeneity in preferences for contracts and preferences for firms, and on the possibility of fractional annuitization, in order to focus on the differences the demand function induced by different institutional regimes. We will return to these concerns below, in the context of demand estimation.

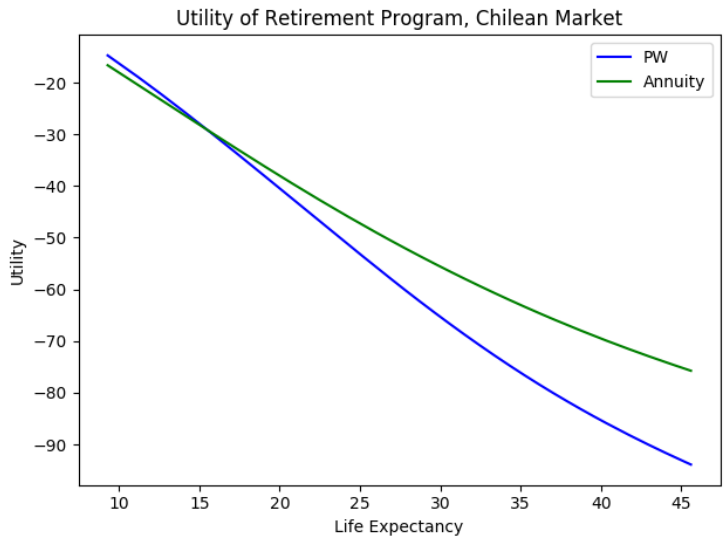
Figure 8 shows the utility levels obtained from retirees choosing an annuity vs. programmed withdrawal. The x axis corresponds to different values of life expectancy after retirement, while the y axis is in utility space. Retirees with a very low mortality shifter, meaning their probability of death is significantly higher than their calendar age, would prefer to take programmed withdrawal over an annuity. The benefit of programmed withdrawal is that retirees get larger payouts in the first few years after retirement than they would receive from an annuity, and the remainder of the savings is passed on to the retirees' beneficiaries. The value of programmed withdrawal is comparable to that of an annuity at all mortality shifters, but of course

[20] Superintendencia de Pensiones and Superintendencia de Valores y Seguros

[21] These tables are specifically designed to capture the mortality expectations of the annuitant population.

[22] Wealth in the pension system is 2200 UFs, and outside wealth is 8800 UFs

Figure 8: Comparison of utility levels from annuity purchase vs programmed withdrawal, given private information about life expectancy as shown on x axis.

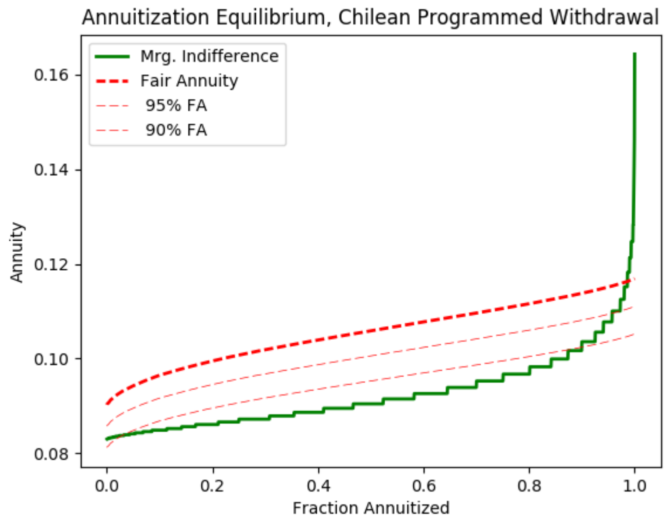


as life expectancy increases the annnuity dominates.

Given the utility levels at each mortality shifter, demand can be derived by imposing a distribution over mortality shifters. Demand is calculated as the monthly payout from an annuity that makes the marginal consumer indifferent between taking an annuity and programmed withdrawal. We call this value the ”indifference annuity”. The result is plotted in figure 9. The x axis shows the fraction of the population purchasing an annuity, and the y axis shows the value of the indifference annuity. The green line, labelled “Marginal Indifference”, denotes the demand function, while the solid red line, labelled ”Fair Annuity”, is average cost. Note that demand is upward sloping on these axes, since a higher pension payout is equivalent to a lower price for a standard good. Therefore, the first individuals to annuitize are those with the lowest indifference annuities, as they would be willing to accept the least generous offers (highest prices). And as the fraction of the population that annuitizes increases, the indifference annuity increases as well: marginal annuitants value annuities less and less, so they need higher payouts (lower prices). From the same parameters, we can also calculate average cost given an annuitant population as the pension payout that lets the firm break even (hence “fair annuity”). Since the only source of heterogeneity in this calibration is mortality risk, individuals with the highest valuation for annuities are also the longest-lived, and therefore the costliest. As a result, the annuity payout that lets the firm break even also increases as more individuals annuitize, as the marginal annuitant is always shorter lived than the inframarginal annuitants.

The intersection of the average cost curve and demand describes the market equilibrium under perfect competition. At the equilibrium in figure 9, annuitization rate is about 99%. As this is under perfect competition, there is no load over the average cost for the annuitized fraction of the population. The dotted lines show the effect of adding a load of 5 or 10%, which results in a lower pension payout to retirees. The

Figure 9: Demand for annuities as a function of markup over actuarially fair annuity for mean individual relative to average cost to firm of insuring that population implied equilibrium annuitization rate of 99% assuming zero load, with 97% annuitization with 15% load.



load may be due to administrative costs or to market power that allows life insurance firms to make positive profits. Figure 9 shows that a load of 5 or 10% decreases the annuitization rate very little, with the new equilibrium annuitization rate being about 90%. This is due to demand being locally inelastic: near full annuitization, the marginal consumers have a very low valuation for annuities and therefore must be offered very high annuity payments. The steepness of the demand curve around full annuitization implies that adding a load doesn't shift the fraction annuitized in an economically significant amount. In this calibration, the high annuitization equilibrium in Chile is very stable in the face of potential supply side changes.

To contextualize these calibration results, we construct a counterfactual equilibrium in the presence of US-style regulations. Following Mitchell et al. (1999), we model Social Security as an actuarially fair annuity provided for half of the individual's pension savings, while the other half is unconstrained and may be annuitized in the private market. Figure 10 compares the utility obtained from annuitizing the remaining wealth ("Annuity") with the utility obtained from keeping the remaining wealth liquid. Figure 11 presents the results of the same supply-and-demand analysis as before. Average cost (per-dollar annuitized) remains the same, as the mortality distribution has not changed. Note, however, that the existence of Social Security significantly changes the demand function, as now retirees value annuities less and the demand function is shallower. Since 50% of pension wealth is already annuitized, exposure to longevity risk is significantly lower, lowering willingness to pay and smoothing out its differences across retirees. Despite these changes, in the no-load scenario we still see almost full annuitization[23], as in the Chilean equilibrium.

[23] While demand and average cost intersect twice, only the higher intersection is an equilibrium because average cost must intersect demand from below in order for the firm to break even.

Figure 10: Comparison of utility levels from annuity purchase vs US-style social security, given private information about life expectancy as shown on x axis.

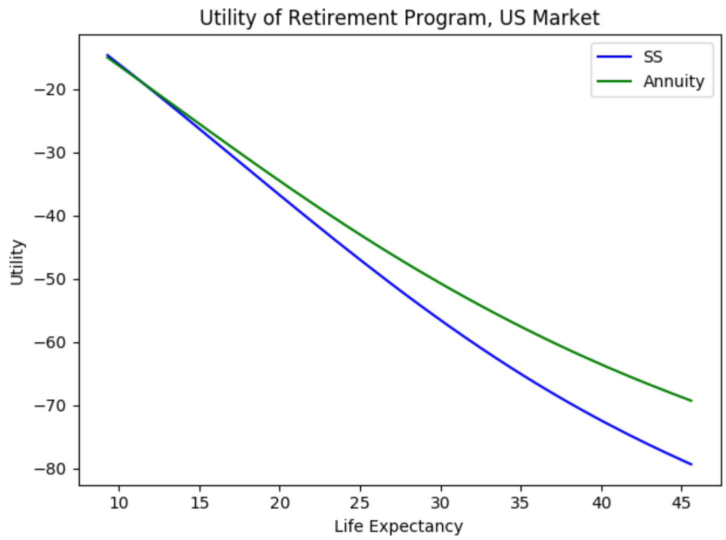
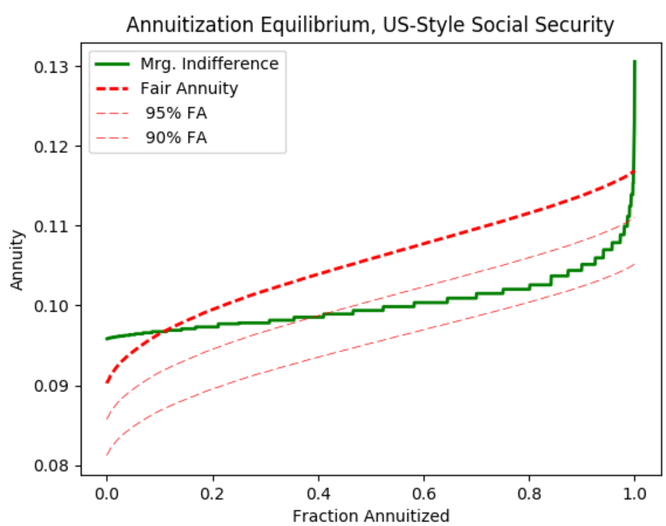


Figure 11: Demand for annuities as a function of markup over actuarially fair annuity for mean individual relative to average cost to firm of insuring that population implied equilibrium annuitization rate of 99% assuming zero load, with 0% annuitization with 15% load.



Once a load is added, though, the US-style equilibrium changes very significantly. In fact, a 10% load causes the entire market to unravel. This is due to the combined effects of the shift up in the indifference annuity, as well as the flattening of the demand function. Therefore, in the US setting adverse selection has a stronger effect on demand for annuities, since Social Security provides a more comparable product to full annuitization than the programmed withdrawal outside option. Then, the regulatory environment of US-style Social Security causes the annuity market equilibrium to be much more sensitive to loads. Any market power or administrative cost has the potential to completely unravel the equilibrium.

The calibration shows that Chile's unique exception to the annuity puzzle could be driven in part by the design of programmed withdrawal, relative to US-style Social Security. In addition, the calibration shows that incorrect measurements of loads and other supply side parameters can significantly change outcomes. Therefore, we need to estimate demand and average cost in a way that makes the minimum number of assumptions about supply of annuities. In addition, we have shown that both demand and average cost are nonlinear, which has a significant impact on equilibrium outcomes. This implies that in order to estimate the differential level of selection in Chile and its contribution to the high annuitization rate, we cannot rely on linear approximations to the demand and cost curves. That approach, which is successful in calculating welfare and counterfactuals in contexts like health insurance (Einav et al. (2010)), would not capture the relatively sudden unraveling of the US market relative to the Chilean market. In the following sections, we proceed to identify the underlying distribution of private information that drives both demand and cost curves. This will allow us both to simulate the US equilibrium and to perform welfare comparisons across pension regimes.

6 Demand Estimation

In this section, we embed the numerical solutions to the model introduced in Section 4 into a demand estimation framework to recover distributions of unobserved preferences. We will then use these estimates in Section 7 to study the impact of different features of the Chilean retirement exchange on equilibrium outcomes. This section is divided into three parts. The first presents the demand estimation model, the second discusses implementation details, and the third discusses identification.

6.1 Framework

In the previous section, we showed how to obtain the value of any offer in the system given the characteristics of the contract, individual observables such as age and gender, and individual unobservables such as initial wealth, risk aversion, bequest motive and mortality and bankruptcy beliefs. Denote the set of individual-offer-firm observables that enter into the optimal consumption-savings problem when faced with an annuity offer as X^A_{ioj}, the analogous individual-firm set for a PW offer as X^PW_{ij}, and individual i's combination of unobservables - a "type" - as θ_i. We can then denote the value of an annuity offer o that firm j makes to individual i by $V^A(X^A_{ioj}, \theta_i)$, and the value of taking programmed withdrawal from PFA j as

$V^{PW}(X_{ij}^{PW},\theta_i)$.

To be more precise about what enters into these objects, age and gender are individual observables that affect the utility calculation for both annuities and PW, as individuals retire at different ages and there are significant mortality differences across genders. For annuities, the payment amount, deferral and guarantee periods, and payments upon bankruptcy ρ_{oj} also enter into the problem; we also match firm risk ratings to Fitch Ratings' 10 year Average Cumulative Default Rates for Financial Institutions in Emerging Markets for 1990-2011 and use these rates as bankruptcy probability beliefs. For PW offers, individuals also need to take into account the fee. As for types, the following unobservables enter the problem: risk aversion γ_i, outside wealth ω_i, bequest motive β_i, and mortality probability vector μ_i. We will denote the joint distribution of these unobservables as $F(\theta)$.

Our goal is to recover the joint distribution $F(\theta)$. To do so, we apply the estimator developed by Fox et al. (2011) and Fox et al. (2016). First, we discretize the space of types and solve the optimal consumption-savings problem for every individual-offer-type combination. Then, we assume that each individual-type combination selects the highest utility offer available to them, and solve for the joint distribution of types that rationalizes observed choices. To be more specific, denoting a point in this grid by r, we impose that the probability a individual i accepts offer o from firm j if they are of type r when faced with the set of annuity offers $\mathscr{O}_i^A$ and the set of PW offers $\mathscr{O}_i^{PW}$ is

$$s_{iojr} = \begin{cases} 1 \text{ if } V^A(X_{ioj}^A,\theta_r) \geq \max[\max_{o',j' \in \mathscr{O}_i^A} V^A(X_{io'j'}^A,\theta_r), \max_{j' \in \mathscr{O}_i^{PW}} V^{PW}(X_{ij'}^{PW},\theta_r)] \\ 0 \text{ otherwise} \end{cases} \tag{16}$$

We then estimate the type probabilities that rationalize observed choices using constrained OLS:

$$\min_{\pi} \sum_{i,o,j} (y_{ioj} - \sum_r s_{iojr}\pi_r)^2 \tag{17}$$

$$\text{subject to:}$$

$$\pi_r \geq 0 \forall r$$

$$\sum_r \pi_r = 1$$

where $y_{ioj} = 1$ if individual i accepts offer o from firm j and 0 otherwise.

There are two important considerations to take into account when considering this model. The first is the choice of grid, an issue we will discuss in detail below. The second is the implications of the assumption that each type accepts the offer that maximizes the value obtained from the consumption-savings problem. This implies that, conditional on a contract type, the only source of heterogeneity across firms beyond the amount they offer is their bankruptcy probability: there can be no non-financial utility components. Thus, the model cannot rationalize the acceptance of dominated offers. While 19% of accepted annuity contracts are dominated, the small monetary amounts lost when accepting a dominated offer - around 1% of the offered amount - leave us unconcerned by this feature of the model. Below we present robustness checks restricting the sample to individuals who do not accept a dominated offer.

Another implication of this assumption is that we are also assuming away the standard endogeneity concern in demand estimation by ruling out non-financial utility terms that can priced into contracts - again, since the main variation in contract values comes from the contract terms, and not from variation in offered amounts, we do not think that this assumption is restrictive. Furthermore, our counterfactuals of interest analyze cases where there is a perfectly competitive annuity market, and thus do not require the identification of tastes for firms. Finally, one could be concerned about information revelation in the request stage - if individuals with different expected costs request different contracts, then firms should price based on the request phase, creating correlation between the observed offers and the unobserved types. To study the impact of this issue, we present robustness checks restricting our estimation sample to individuals with the same observables and the same requested set of contracts.

6.2 Implementation

The goal of the estimation procedure is to recover the joint distribution of unobserved preferences without specifying restrictive functional form assumptions. We will now discuss the details of the current implementation.

First, we need to pick a grid over the space of π. Recall that $F(\pi)$ is the joint distribution of risk aversion γ, initial wealth ω, bequest motive β and mortality probability vector μ. These objects must be further restricted, or creating a grid over this space will be computationally infeasible (μ alone is a $T \times 1$ object). First, we model the mortality probability vector μ as the mortality vector from the Chilean mortality tables in place at the time of retirement plus an unobserved shifter that makes retirees an arbitrary number of years younger or older than their retirement age. For example, an individual who retires at 60 with a mortality shifter value of 2 solves the optimal consumption-savings problem for each contract using the mortality vector of a 62 year old. This allows the model to continue to feature adverse selection into contracts, as individuals with low (high) mortality shifter draws are unobservably younger (older) than their age, without having to separately identify whether this selection comes from a higher death probability in year x or $x+1$.

This shrinks the dimensionality of the grid to 4: risk aversion, initial wealth, bequest motive, and mortality shifter. We solve the optimal consumption-savings problem for every individual-firm-annuity offer, imposing $\delta = 0.95$, and $R = 1.03$. For programmed withdrawal, since PW fees are almost identical across PFAs and we are not interested in modelling substitution across them, we solve the optimal consumption-savings problem for one PW offer, assuming the fee is the median fee. Furthermore, we assume that the PW problem is non-stochastic ($\sigma_{PW} = 0$), and set the mean PW return to its empirical counterpart.

As discussed earlier, the choice of grid is a critical input into the estimation procedure. Instead of selecting the grid arbitrarily, we incorporate a model selection step. To estimate this step, we pre-solve the consumption-savings problem for every offer received by a 5% subsample of retirees and for 17 grid points per dimension of the type space, which corresponds to 83,251 total points. We select these points using a Clenshaw-Curtis grid (Trefethen (2008)), see Appendix table 4 for the gridpoint values. For bequest motive

β, the grid is logarithmic and spans from 0 to 7.88e03. To build intuition, an individual with risk aversion coefficient equal to 3 who knew thet will be dead tomorrow would consume all their wealth today if their bequest motive were 0, while they would consume 5% of their wealth today if $\beta = 7.88e03$. Table 5 in the Appendix presents this mapping for every point in the grid of bequest motives. For risk aversion, the grid spans from 0 to 10, while for outside wealth the grid spans from 0.2 to 20 thousands of UFs.[24] Finally, the mortality shifter grid spans from -15 to 15. Recall that a retiree aged x with a mortality shifter value of y solves the optimal consumption savings problem using the mortality probabilities of an $x+y$ year old. We then estimate the constrained OLS model in Equation 17 and select the types with mass greater than $2e-04$ - this corresponds to 194 types. We then solve the consumption-savings problem for every offer received in the data for each of these 194 types, and re-estimate the constrained OLS problem for the full dataset. For this estimation step, we split the sample by gender and quartiles of pension savings, estimating separate type distributions for each bin of observables.

Note that this model selection step is equivalent to solving the following simplex-constrained Lasso model:

$$\min_{\pi} \sum_{i,o,j} (y_{ioj} - \sum_{r} s_{iojr}\pi_r)^2 + \rho \cdot ||\pi||_1 \tag{18}$$

subject to:

$$\pi_r \geq 0 \forall r$$

$$\sum_r \pi_r = 1$$

as substituting the constraint that coefficients must add to one into the objective function makes the Lasso penalty term irrelevant for optimization.

6.3 Intuition for Identification

How is the distribution of types identified? Following the intuition in Kasahara and Shimotsu (2009), note that the model maps each combination of offer and unobservable type to a (degenerate) choice probability. Let S denote the matrix containing these choice probabilities, where columns are types and rows are contracts. The first order conditions for the constrained OLS estimator imply that π is identified if the matrix $S'S$ is invertible. Loosely speaking, this requires that different types make different choices when faced with the same annuity contract offers. The features of our setting make this likely, as the financial value of an annuity contract is greatly dependent on the match between the contracts' terms and the preferences of the annuitant. For example, as the number of guarantee periods increases, annuity payouts always decrease. This implies that individuals with no bequest motive will always prefer contracts without guarantee periods, while as bequest motive increases retirees will value contracts with longer guarantee periods more. As another example, contracts with deferral years imply a tradeoff between higher annuity payouts until death

[24]In December 12, 2017, the dollar equivalent range was between 8,170 and 817,000

and an initial period of time without any annuity income. Only individuals who expect to live long enough to recoup this investment and who have sufficient assets outside the system to fund the initial periods will find these offers attractive.

There are two concerns with this argument. The first is the assumption that there is no non-financial utility in the observed offers. If that is not the case, then firms may price on these non-financial terms, creating dependence between the observed offer characteristics and the error term. We believe that the empirical relevance of this concern is minor, as the amount of money lost when an individual accepts a dominated offer (one that can only be rationalized by non-financial utility) is small. The second is that the observed variation in offers across individuals is correlated with the distribution of types, by firms screening on observables. To address this concern, we estimate type distributions separately by gender and by quartiles of pension balances, as we expect companies to price on these variables. The remaining observables transmitted to firms before they formulate their offers are age, which is controlled for flexibly in the model, number of legal beneficiaries, for which there is no variation in our subsample, and contracts types requested.

7 Results

This section presents demand estimation results and measures of in and out of sample fit.

7.1 Marginal Distributions of Heterogeneity

Appendix tables 6 through 13 present the 10 grid points with highest mass for every gender - pension balance quartile, while Appendix figures 14 through 21 present marginal distributions for every dimension of unobserved type - gender - pension balance quartile combination.

There are several interesting features of these results. Consider first the marginal distributions of bequest motives (figures 14 and 15). There is striking heterogeneity in this dimension, with a large mass point at 0 for every bin of observables and significant mass above 3.3, which corresponds to the level where an individual with risk aversion equal to 3 would leave half their money to their heirs if facing certain death. Individuals with high bequest motives value mandatory annuitization less, even if it comes at an actuarially fair price. The policy is likely to significantly decrease the utility of this population as well as greatly reduce their demand for annuities. We will return to this issue below. Moreover, women have significantly higher bequest motives than men, a result consistent with findings from the development economics literature on effects of old-age pensions on extended families (Bertrand et al. (2001), Duflo (2003)).

Turning our attention to the marginal distributions of health age, figures 16 and 17 present these objects for women and men, respectively. Recall that an individual who retires at age x with a health shifter of y solves the optimal consumption-savings problem with the mortality expectations of an $x+y$ year old according to the Chilean mortality tables. Therefore, negative values of the health shifter imply that the individual has beliefs that are consistent with being younger (in mortality probability) than their retirement age. Again, there is substantial heterogeneity in this dimension, with between 25% and 45% of women and 20% to 30% of males holding lower mortality expectations than those of their retirement age. Furthermore, both men and women in higher pension balance quartiles have lower mortality probabilities. Recall that the Chilean mortality tables do not vary by wealth, only by gender, so this result is intuitive.

Figures 18 and 19 present the marginal distribution of outside wealth for women and men, respectively. The support of these distributions ranges from $187,910$ dollars to $567,406$ dollars, which seems reasonable when considering that this object is meant to capture the value of all assets that can fund consumption and inheritance, and that our sample is restricted to individuals who can fund an annuity offer above the minimum pension. Reassuringly, the distribution of outside wealth for both genders shifts to the right for individuals with higher pension quartiles.

Finally, figures 20 and 21 present the marginal distribution of risk aversion for women and men, respectively. It is interesting to note that levels of risk aversion are low (most mass is at $\gamma = 0.84$ and $\gamma = 1.46$), and that there is substantially less heterogeneity in this variable than in all other dimensions of unobserved type. Furthermore, women seem to be less risk averse than men, although values for both are low.

7.2 Key Correlations

In this setting, marginal distributions do not tell the whole story, as the relationship between the unobservables will greatly affect choices and equilibrium outcomes. In particular, if individuals with low mortality expectations also have high preferences for annuities due to other unobserved characteristics, then the annuity market can feature advantageous, not adverse, selection. A nice feature of our estimation procedure is that we are non-parametrically estimating the joint distribution of unobserved types, allowing us to flexibly determine the relationship between these unobservables. Tables 14 through 21 in the Appendix present correlation matrices all dimensions of unobserved type.

Bequest motives are negatively correlated with risk aversion for both genders, except in the lowest wealth quartile, and negatively correlated with the health shifter, particularly for women and low wealth men. That is, individuals with higher bequest motives are also cheaper to insure, but have lower willingness to pay for insurance. The correlation between bequest motives and outside wealth is positive for women in quartiles 1 through 3 and men in the second and third quartile, and close to zero for all other groups. Risk aversion is negatively correlated with outside wealth for all groups, and also negatively correlated with health shifter for the first quartile of pension balances for both genders. This is intuitive as higher outside wealth means that individuals can self-insure. However, as pension wealth increases, this correlation gets closer to zero. Finally, there is a positive correlation between outside wealth and health shifter, particularly for lower pension balance quartiles.

These correlations can play significant roles in determining the equilibrium outcomes of the annuity market and of any counterfactual simulation. In a setting where mortality realizations are independent of all other dimensions of type, willingness to pay for an annuity (conditional on gender, age and pension balances) solely depends on these expectations.[25] As a result, the first annuitized dollars in this market will correspond to individuals who expect to be long lived. If these expectations are correct, this will induce adverse selection. However, if other dimensions of unobserved type are correlated with mortality, this result can be overturned. For example, our demand estimates imply that higher bequest motives are associated with longer life expectancies. As a result, long lived individuals have less of an incentive to annuitize, as the payoff to holding liquid assets upon death is greater. This pushes the annuity market towards advantageous selection. As a counterexample, a negative correlation between risk aversion and health shifter - or positive correlation between risk aversion and life expectancy - implies that the individuals who have the greater disutility of outliving their money are also the most expensive to annuitize. We return to the issue of advantageous versus adverse selection into annuitization below.

7.3 Model Fit and Equilibrium

Before discussing these issues, Table 22 presents various measures of in-sample and out-of-sample fit of the demand estimation model, by gender-pension balance quartile. The model does a reasonable job fitting the observed choices, with an R^2 of around 0.5, despite ruling out any non-financial value of an annuity offer. However, it under-predicts both the fraction of retirees who accept an annuity offer and, in particular, it does so by under-valuing annuity offers with a guarantee period. We believe that this is the case because of the asymmetric role sales agents play in the Chilean market, as all of these agents are employees of life insurance companies and not of PFAs. As a result, these agents have an incentive to push for annuity offers more than for programmed withdrawal offers, generating this wedge that our purely financial model cannot overcome. Moreover, in conversations with industry experts we have learned that sales agents typically push guarantee periods, highlighting the financial "loss" from dying soon after annuitization. As a result, we think that this under-prediction is to be expected from any purely financial model. Nevertheless, we are reassured by the fact that the model fits the (out-of-sample) two-year mortality well, which suggests that we are in fact recovering reasonable estimates of the distribution of unobserved types from our model.

We now turn to the analysis of the annuity market equilibrium. Our goal is to analyze the implied annuity demand and average cost curves in the Chilean setting, and to then study how these functions are affected by changing the institutional setup towards the US system. To do so, we analyze the equilibrium of a perfectly competitive annuity market where retirees can distribute their pension savings, dollar for dollar, into either an immediate annuity with no guarantee period or programmed withdrawal (with no minimum pension guarantee). Our goal is to show that even in this basic setting changing the rules of the system to make Chile more like the US leads to market unravelling. Note that since age, gender and pension balances can be priced on, every combination of these quantities constitutes a separate market with potentially different equilibria. In what follows, every equilibrium that is presented will be conditional on a combination of these quantities.

Figure 22 presents the implied annuity demand curve (green) and average cost curve (red) for a 60 year

[25] Which is the relevant conditioning set of variables in Chile, as all of these quantities are observed and priced on

old female retiree that retires with a pension balance equal to the first, second, and third quartiles of the distribution of pension balances for females during our sample period. We construct the annuity demand curve by choosing a grid over the space of yearly annuity payouts (expressed as a percentage of the pension balance) and, for every type and payout, solving for the optimal allocation of pension balances between PW and an annuity. This gives us the fraction of wealth that each type would annuitize across the grid of annuity payouts. We then aggregate to the market level using the estimated type probabilities, and plot the curve relating annuity payouts to the aggregate fraction of wealth annuitized - the demand curve in this setting. This curve can also be interpreted as a "willingness-to-accept" curve, expressing the worst annuity offer that induces the marginal annuitized dollar. We calculate the average cost curve by calculating the expected cost of a unit annuity for every type, and then using the fraction of wealth each type annuitizes given a price and the type probabilities to obtain the expected cost for the annuitant population at each price. We express this expected cost in the same units as the demand curve, so that it can be interpreted as the break-even annuity offer given the expected cost of the annuitant population. Under adverse selection, this curve is upward-sloping: dollars that are only annuitized after high offers correspond to individuals with higher mortality. Under advantageous selection, on the other hand, the curve is downward sloping. Under our framework, the average cost curve can have regions where advantageous selection and regions of negative selection, depending on the relative mortality expectations of the marginal and inframarginal annuitants. In figure 22, we find that the equilibrium for women hovers slightly below 50% annuitization, with annuity offers near 5.25% of the pension balance. Moreover, we find regions of advantageous selection around the market equilibrium. Finally, we find that the annuity demand curve is extremely non-linear, reflecting the significant heterogeneity in the distribution of types found earlier. Results for 65 year old men (figure 23) exhibit significantly higher annuitization rates and significantly better offers, although part of this result is mechanical: 65 year old men have higher mortality rates than 60 year old women, and as a result are a lower cost population. However, the degree of advantageous selection near the market equilibrium for men seems to be higher for men than for women, which suggests that not all of this discrepancy is mechanical.

In fact, the impact of adverse selection on the fraction of wealth annuitized for women (figure 22) is quite large: at the actuarially fair price for the whole population, equilibrium annuitization would rise to around 70% - 80% of available funds [26]. Furthermore, the equilibrium price is between 93% and 96% of the actuarially fair price for the whole population. A similar result holds for first quartile males, for whom the equilibrium price is 93% of the actuarially fair annuity. However, for males in the second and third quartile of pension balances the average cost curve is mostly flat, such that the equilibrium annuity offer is equivalent to the actuarially fair annuity if the whole population annuitizes. While there has been plenty of debate in Chile about discrepancies in pension payments between men and women, most of it has focused on retirement age and differences in mortality tables. We believe that this result is the first instance that documents that differential selection leads to worse annuity offers for women than for men in the Chilean market.

[26] The Fair Annuity curve at 100% is the actuarially fair price. Projecting this value until the intersection with the annuity demand curve yields this result

It is important to note that the preceding figures are not a good metric for quantifying the fit of the model, as we have removed several features of the Chilean system in order for these equilibria to be a reasonable benchmark for the counterfactuals that follow. In particular, we are removing the minimum pension guarantee, as well as programmed withdrawal commissions. Finally, we are also focusing on a single contract type, and not allowing for differentiation across firms. These assumptions simplify the problem and allow us to focus on the shifts in demand and average cost induced by the rules governing how funds can be accessed.

7.4 Social Security Counterfactual

We now reform the Chilean institutional setup by introducing Social Security and by replacing programmed withdrawal with the ability to withdraw retirement savings in a lump sum, with the goal of determining how the institutional setup of the Unites States affects the annuity market equilibrium. Following Mitchell et al. (1999), we begin by modeling Social Security as a 50% tax on pension balances that is returned to retirees via an actuarially fair annuity. Retirees then decide to allocate their remaining funds, dollar for dollar, to either a private-market annuity and to a lump-sum withdrawal. Figures 24 and 25 present the results of this exercise for women and men, respectively. In all markets, this reform induces a significant contraction of the demand curve, which leads to significantly worse equilibrium annuity contracts - even full market unraveling in the case of second quartile females. It is interesting to note how the response of the annuity market equilibrium varies across genders: while for women the highest annuitization rate is between 0% and 20% in the US counterfactual, male annuitization rates are between 50%-70%.

There are two main forces leading to these results. First, lump sum withdrawals weakly dominate programmed withdrawal, as a lump sum has identical bequest and risk properties as PW but does not constrain the path of consumption over time. Second, mandatory annuitization of 50% of pension balances immediately eliminates the risk of outliving one's savings, turning additional annuitization into "top-up" insurance. This reduces willingness-to-pay across the population, contracting the demand curve. It also reduces heterogeneity in willingness to pay, flattening the demand curve, as types with different levels of outside wealth have significantly different probabilities of hitting zero consumption in the Chilean equilibrium when not annuitizing, but the same probability (zero) once a fraction of pension balances are mandated to be annuitized.

To unpack these effects, we repeat the previous exercise for different levels of the fraction of pension wealth that is assigned to Social Security, ranging from 0% to 90%. That is, we find the equilibrium fraction of remaining wealth that is annuitized when retirees have x% of their wealth annuitized at the actuarially fair price, and when the remaining 1-x% can be allocated between an annuity and a lump sum withdrawal. Figures 26 and 27 plot the equilibrium fraction of wealth annuitized from this exercise, for women and men, respectively, while Table 23 reports the equilibrium annuity payouts. Consider the case where 0% of funds are allocated to Social Security. The only difference between this setting and the Chilean equilibrium is the outside option, which in this case is lump-sum withdrawal. Despite the fact that lump sum withdrawals are a more appealing product than programmed withdrawal, which leads to a contraction in demand for annuities, the equilibrium in these cases is always interior. For women in all three pension quartiles, and men in the first quartile, the equilibrium annuity payout is lower in this case than in the Chilean equilibrium. For example, Panel A in table 23 reports that the Chilean equilibrium annuity for women in the second quartile of pension savings pays out 5.52% of the account balance per year, while the 0% Social Security equilibrium annuity pays out 5.31%. Therefore, in these cases changing the outside option from programmed withdrawal to lump sum withdrawal benefits those who would fully allocate their pension savings to PW, as they now can now access a product with fewer constraints on the consumption path, but it harms those who fully annuitize, as they now receive worse payouts.

As the amount in Social Security increases, we can see that for these populations the equilibrium annuity gets progressively worse. However, since the actuarially fair annuity is greater than the US equilibrium annuity, on net an individual who fully annuitizes in the US receives higher annuity payouts once Social Security kicks in. For example, Panel B in table 23 reports that in the 25% Social Security equilibrium a woman in the first quartile who fully annuitizes receives a payout of 5.32%, while the Chilean equilibrium would pay out 5.26%. That is, barring market unraveling, individuals who prefer to fully annuitize in Chile are better off in the US equilibrium than in the Chilean equilibrium once the fraction of wealth allocated to Social Security is sufficiently high. Once market unraveling occurs, as is the case for women in the second and third pension quartiles, the welfare comparison between Chile and the US for this subgroup of retirees is ambiguous: on the one hand, they receive a higher annuity payout for a fraction of their wealth, but on the other, they are forced to withdraw the remainder in a lump sum, which they value less than the Chilean equilibrium annuity. The story is more clear cut for those who hold all their money in PW in the Chilean equilibrium: increases in the amount of money in Social Security represent more exposure to an asset that they value less than programmed withdrawal, and less exposure to the lump sum. On net, once the fraction of wealth in Social Security is sufficiently high, they must be worse off. Therefore, for women in all three pension quartiles, and men in the first quartile, it is not immediate whether the move from the Chilean equilibrium to the US equilibrium is welfare increasing or not.

For men in the second and third quartiles, however, moving from the Chilean equilibrium to the 0% Social Security setting is Pareto improving, as the equilibrium annuity offer is weakly greater in the US 0% case than in Chile, and lump sum withdrawals weakly dominate PW. The former result stems directly from advantageous selection: a contraction in the demand curve leads to higher annuity prices, as higher cost individuals are marginal. As another consequence of advantageous selection, the actuarially fair annuity is worse than the 0% equilibrium annuity. For these subgroups, increasing the amount of wealth allocated to Social Security leaves the equilibrium payout mostly flat and never makes the market unravel, although the effective annuity payout falls as exposure to the actuarially fair annuity increases. Still, payouts under the Social Security equilibria are always higher than in the Chilean case, so those who would fully annuitize in Chile are strictly better off in all the US equilibria. However, retirees who allocate all their money to PW are progressively worse as the fraction of wealth allocated to Social Security increases, as before. Interestingly, for men in the second and third quartiles the US 0% equilibrium Pareto dominates all others, as the types who value annuitization get the highest annuity payout while those who do not enjoy fully unrestricted withdrawals.

7.5 Welfare

Analyzing the equilibirum outcomes allow us to sign the welfare effect of moving from the Chilean equilibrium to the US equilibrium for certain types in the population. The next step is to quantify the welfare comparison between the Chilean and US systems. To do so, we calculate for each type their compensating variation, or the transfer into their US pension balance that would make them indifferent between retiring in the US or in Chile. More precisely, define $V^*_{CHILE}(W_0, \theta_r)$ to be type θ_r's utility derived from their optimal allocation of pension balance W_0 across programmed withdrawal and an annuity at the Chilean equilibrium price, and define $V^*_{USx\%}(x\% \cdot W_0, (1-x\%) \cdot W_0, \theta_r)$ to be type θ_r's utility derived from their optimal allocation of remaining pension balance $(1-x\%) \cdot W_0$ across lump sum withdrawal and an annuity at the US x% equilibrium price, taking into account that $x\% \cdot W_0$ is returned to them through an actuarially fair annuity. Then the compensating variation for type θ_r between the US x% equilibrium and the Chilean equilibrium solves:

$$V^*_{CHILE}(W_0, \theta_r) = V^*_{USx\%}(x\% \cdot W_0, (1-x\%) \cdot W_0 + CV(x\%, \theta_r), \theta_r) \tag{19}$$

The left-hand-side panels of figures 28 and 29 plot the cumulative distribution functions of compensating variation for women and men, respectively, for different values of the fraction of wealth in Social Security. The right-hand-side panels of the same figures plot the average CV for each US equilibrium. The CDF plots consistently show significant heterogeneity across the population, with significant masses in both the space where retirees prefer Chile to the US and in the space where the converse holds. For example, the Female Q2 50% Social Security distribution exhibits both retirees who are willing to pay up to $20,000 dollars to move from the Chilean equilibrium to this US equilibrium and individuals who would have to be given at least $10,000 dollars to be willing to make this move. For this submarket, the average compensating variation is close to zero. The only case where the US consistently delivers higher welfare than Chile across submarkets is the 0% Social Security, where only a very small mass of retirees actually prefers Chile. This is the case because individuals who fully invest in PW are immediately better off, as lump sum withdrawals dominate PW, while the change in annuity rates is so small that individuals who prefer to fully annuitize are mostly indifferent. The average CV plots consistently show that this quantity is increasing in the fraction of wealth in Social Security, although there is heterogeneity across submarkets in whether the average CV ever becomes positive.

To unpack these results, it is useful to divide retirees according to their choices in the Chilean system. In particular, consider those who fully annuitize in Chile, those who fully take up programmed withdrawal, and those who mix between both options. Table 24 reports the fraction of consumers in each of these categories in the different submarkets analyzed. For retirees who always annuitize in Chile, a sufficient condition for a US equilibrium to dominate the Chilean equilibrium is that full annuitization is available in the US at a better rate[27]. This holds in the Female Q1 submarket when x% is 25% or higher, in the Female Q2 case for 25% Social Security, in the Female Q3 case for 25% and 50% Social Security, in the Male Q1 case for x% 75% and higher, and in all cases for males in Q2 and Q3. Furthermore, in these cases as the gap between the Chilean Equilibrium rate and the US full annuitization payout increases, this group of retirees prefers the US more, and their CV shifts to the left. As this group represents between 35% of retirees (Female Q3) and 76% of retirees (Male Q3), they compose most of the left tail of the distribution of compensating variation in these cases.

If this sufficient condition does not hold, either because of pricing or because the US full annuitization rate is below the Chilean equilibrium price, then the welfare comparison for retirees who fully annuitize in Chile is ambiguous. In all the 0% cases where this condition does not hold (all female cases and male Q1), these retirees are better off in Chile. However, as price differences as small in all but the Male Q1 case, their CVs are very close to zero. In the male Q1 case, this group is responsible for the mass above 0. More interesting are the Female Q2 and Q3 cases where the market fully unravels, as in these settings it is not clear whether the high payout of the actuarially fair annuity and the greater value of lump sum withdrawals compensates for the lack of an annuity option for the remaining wealth. Interestingly, it turns out that it does: Female Q2 and Q3 retirees who always annuitize in Chile are better off in all of the unravelled US equilibria. Finally, for Q1 males who always annuitize in Chile the US 25% and 50% cases yield both positive and negative welfare changes, depending on how types trade off the worse annuity price with the increased value of the alternative.

Welfare comparisons are much more clear cut for those types who fully take up programmed withdrawal in Chile. In this case, the US 0% equilibrium is always welfare increasing, as lump sum withdrawals dominate programmed withdrawal. However, as the fraction assigned to Social Security increases, these retirees are forced to hold an annuity that they value significantly below the equivalent lump sum. We find that for Social Security levels at 25% and higher these retirees always prefer Chile and the compensating variation for this group turns positive. Thus, these types comprise a significant fraction of the mass of consumers who prefer Chile to the US, and are responsible for the shift to the right in the right tail of the CV CDF as the amount allocated to Social Security increases. However, different submarkets have significantly different masses of retirees who always choose PW in Chile, ranging from 4.6% for Q3 males to 56% for Q1 females. This variation is responsible for the differences in mass in the right tail across submarkets.

Finally, individuals who mix between annuities and PW in Chile prefer the US 0% equilibria to Chile, as the increase in the value of the outside option overwhelms the difference in prices. However, as the fraction of wealth in Social Security increases, the utility these types perceive from the US equilibrium decreases and they begin preferring Chile: these types become overannuitized in the US. Across the board, types with

[27] If z^{AF} is the actuarially fair annuity and z^{EQ} is the equilibrium annuity, the relevant rate is $x\% \cdot z^{AF} + (1-x\%)z^{EQ}$

interior solutions in the Chilean case will allocate 0% of their wealth to annuities if the fraction of wealth in Social Security is at 25% or higher, suggesting that they value annuities because of the floor placed on their payouts but once that floor is set they value liquidity more. This is consistent with types with high levels of risk aversion and high bequest motives. From a welfare perspective, then, these types behave similarly to the previous group, although their compensating variation amounts tend to be lower.

Therefore, despite the fact that US annuity prices are often worse than in Chile, and that the annuity market can even unravel, the main driver of welfare differences between the Chilean system and a system like the United States' is retirees' underlying preferences for annuitization. That is, retirees with high valuations for annuities prefer a system that delivers an actuarially fair annuity for a significant portion of their wealth even if the remaining wealth cannot be annuitized. That is because the insurance value of the first annuitized dollars is higher. On the other hand, retirees with low preferences for annuitization, such as those with high bequest motives, obtain significantly higher utility from the Chilean equilibrium thanks to the ability it gives them to assign low amounts of wealth to an annuity. This suggests that allowing individuals to either buy out their Social Security annuity or to convert it to programmed withdrawal could be a way to leave all types better off in the United States.

8 Conclusion

The Chilean annuity market has several striking features. First, a large majority of retirees choose to purchase private annuities with their retirement savings. Second, prices of annuities in equilibrium are 3-5% more expensive than actuarially fair, significantly cheaper than the 10-15% markup over actuarial fairness estimated in the US. Third, the outside option to annuitization (PW) is relatively valuable, especially to retirees with low life expectancies or high bequest motives. Fourth, the Chilean market exhibits evidence of adverse selection, as do many other annuity markets. Finally, calibrations show that demand for annuities in Chile is more price inelastic and less sensitive to loads than demand for annuities in a US-style regulatory regime.

The evidence laid out in the reduced form facts and calibrations imply that, under certain parameters, Chile's annuity market may unravel if reforms are introduced that move Chile towards a US-style social security system. Furthermore, the shape of the demand and average cost curves found in the calibration exercise make it clear that simple reduced form estimation approaches are not well suited to determine whether the market will unravel under counterfactual regulatory regimes or to study the welfare implications of these reforms. To tackle this issue, we build a structural model based on the standard consumption-savings problem faced by retirees, that accounts for income effects and other features of nonlinear utility that influence retirees' choices. The aim is to estimate the distribution of unobservable retiree characteristics from which we can derive their demand for annuities and the cost firms face to insure them. The model adapts the methodology developed by Fox et al. (2011). We pre-solve this lifecycle model for a representative set of consumers over a wide variety of combinations of unobserved preferences. We then estimate the distribution of weights on the distribution of consumers that allows us to match moments observed in the data.

Demand estimates show that individuals have significant and highly varied private information about their own mortality. They also have heterogeneous bequest motives, with women having higher bequest motives than men - or a higher willness to pay for products that provide some value to their heirs. We also find significant unobserved heterogeneity in mortality expectations relative to the Chilean mortality tables, with poorer individuals having higher mortality probabilities.

Armed with these demand estimates, we simulate market equilibria under stripped-down versions of the Chilean and US institutional frameworks. Our goal is to highlight the change in demand and average cost induced by the introduction of Social Security. At our estimated parameters, the Chilean equilibrium shows high annuitization, while moving to a US-style social security system contracts demand for annuities and leads to lower annutization rates. In some cases, such a reform can even lead the annuity market to unravel.

Despite the fact that US annuity prices are often worse than in Chile, and that the annuity market can even unravel, the main driver of welfare differences between the Chilean system and a system like the United States' is retirees' underlying preferences for annuitization. That is, retirees with high valuations for annuities prefer a system that delivers an actuarially fair annuity for a significant portion of their wealth even if the remaining wealth cannot be annuitized. That is because the insurance value of the first annuitized dollars is higher. On the other hand, retirees with low preferences for annuitization, such as those with high bequest motives, obtain significantly higher utility from the Chilean equilibrium thanks to the ability it gives them to assign low amounts of wealth to an annuity. This suggests that allowing individuals to either buy out public pensions or to convert it to programmed withdrawal could improve welfare across the board in the United States.

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A Appendix Tables and Figures

	(1)
	Time to Death
Choose annuity	-0.164**
	(0.0601)
Insurance co. agent	0.195**
	(0.0646)
Insurance broker	0.160*
	(0.0682)
Financial advisor	0.0841
	(0.103)
Direct thru insurance co.	0.133
	(0.189)
Wealth/age controls	✓
Observations	45091

Figure 12: Correlation between death hazard and choice to annuitize, Gompertz baseline hazard

Number of Previous Policies	Percentage of Acceptances
0	96.80%
1	2.90%
2	0.20%
3 or more	0.10%

Figure 13: Evidence that majority of retirees have never previously interacted with annuity company (Castro et al. (2018))

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
1	0	0	0.2	-15
2	8.99E-07	0.09	0.39	-14
3	6.07E-05	0.38	0.95	-13
4	7.58E-04	0.84	1.87	-12
5	4.85E-03	1.46	3.1	-10
6	2.20E-02	2.22	4.59	-8
7	8.21E-02	3.08	6.31	-5
8	2.72E-01	4.02	8.17	-2
9	8.52E-01	5	10.1	0
10	2.60E+00	5.98	12	2
11	8.05E+00	6.91	13.9	5
12	2.61E+01	7.78	15.6	8
13	9.06E+01	8.54	17.1	10
14	3.44E+02	9.16	18.3	12
15	1.37E+03	9.62	19.2	13
16	4.63E+03	9.9	19.8	14
17	7.89E+03	10	20	15

Table 4: Gridpoints by dimension of types for initial grid

	Bequest Motive	Percentage Consumed
1	0	100.00%
2	8.99E-07	99.09%
3	6.07E-05	96.38%
4	7.58E-04	91.99%
5	4.85E-03	86.09%
6	2.20E-02	78.90%
7	8.21E-02	70.68%
8	2.72E-01	61.79%
9	8.52E-01	52.50%
10	2.60E+00	43.25%
11	8.05E+00	34.33%
12	2.61E+01	26.10%
13	9.06E+01	18.92%
14	3.44E+02	13.01%
15	1.37E+03	8.62%
16	4.63E+03	5.91%
17	7.89E+03	5.00%

Table 5: Map from bequest motive to fraction of wealth consumed before certain death

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass
1	26.07	0.84	12.03	10	8.92%
2	26.07	0.84	4.60	0	8.92%
3	26.07	0.84	10.10	10	8.88%
4	90.66	0.84	6.31	-2	7.73%
5	7.58E-04	1.46	12.03	-2	7.53%
6	0.27	5.00	10.10	0	7.03%
7	26.07	0.84	8.17	10	7.02%
8	90.66	1.46	8.17	5	5.90%
9	0.85	5.00	8.17	0	5.42%
10	0.85	5.00	12.03	0	3.41%

Table 6: Top 10 Mass Points for Females in the First Quartile of Pension Savings

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass
1	90.66	0.84	6.31	-2	9.76%
2	6.07E-05	1.46	10.10	5	6.72%
3	26.07	0.84	8.17	10	6.55%
4	26.07	0.84	10.10	10	5.82%
5	26.07	0.84	4.60	0	5.69%
6	26.07	0.84	12.03	10	5.50%
7	26.07	0.84	6.31	-2	5.12%
8	7.58E-04	1.46	10.10	-5	5.02%
9	6.07E-05	1.46	8.17	-2	3.84%
10	90.66	1.46	6.31	2	3.78%

Table 7: Top 10 Mass Points for Females in the Second Quartile of Pension Savings

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass
1	26.07	0.84	10.10	10	7.91%
2	26.07	0.84	12.03	10	7.91%
3	90.66	0.84	6.31	-2	6.81%
4	6.07E-05	1.46	12.03	-2	6.53%
5	90.66	1.46	6.31	2	6.10%
6	26.07	0.84	4.60	0	5.97%
7	26.07	0.84	8.17	10	4.59%
8	7.58E-04	1.46	8.17	-5	4.32%
9	344.28	1.46	10.10	-2	4.02%
10	26.07	0.84	6.31	-2	3.43%

Table 8: Top 10 Mass Points for Females in the Third Quartile of Pension Savings

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass
1	26.07	0.84	8.17	10	14.77%
2	344.28	1.46	6.31	-2	11.09%
3	6.07E-05	1.46	12.03	-2	10.68%
4	6.07E-05	1.46	10.10	5	7.69%
5	6.07E-05	1.46	8.17	-2	7.18%
6	90.66	1.46	6.31	2	6.87%
7	26.07	0.84	12.03	-2	6.71%
8	26.07	0.84	6.31	-2	3.87%
9	7.58E-04	1.46	12.03	-5	3.72%
10	8.06	0.84	8.17	5	3.28%

Table 9: Top 10 Mass Points for Females in the Fourth Quartile of Pension Savings

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass
1	0.27	5.00	10.10	0	16.47%
2	8.06	1.46	8.17	8	7.89%
3	26.07	0.84	4.60	0	5.85%
4	90.66	0.84	6.31	-2	4.98%
5	7.58E-04	1.46	8.17	-5	4.07%
6	7.58E-04	1.46	12.03	-5	4.06%
7	7.58E-04	1.46	10.10	-5	4.04%
8	90.66	1.46	8.17	2	3.63%
9	26.07	0.84	8.17	10	2.90%
10	26.07	0.84	10.10	10	2.88%

Table 10: Top 10 Mass Points for Males in the First Quartile of Pension Savings

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass
1	6.07E-05	1.46	10.10	5	21.67%
2	7.58E-04	1.46	6.31	0	7.92%
3	26.07	0.84	4.60	0	7.37%
4	7.58E-04	1.46	8.17	-5	6.60%
5	90.66	0.84	6.31	-2	6.16%
6	0.27	4.02	10.10	2	5.40%
7	0.85	2.22	10.10	8	4.84%
8	90.66	1.46	10.10	5	4.37%
9	2.60	3.09	10.10	0	3.92%
10	344.28	1.46	6.31	-2	2.92%

Table 11: Top 10 Mass Points for Males in the Second Quartile of Pension Savings

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass
1	6.07E-05	1.46	10.10	5	24.68%
2	7.58E-04	1.46	8.17	-5	7.72%
3	26.07	0.84	4.60	0	6.62%
4	344.28	1.46	6.31	-2	6.16%
5	90.66	1.46	8.17	5	4.27%
6	26.07	0.84	12.03	-2	3.64%
7	2.60	2.22	8.17	8	3.42%
8	90.66	1.46	6.31	2	3.34%
9	0.85	5.00	8.17	0	3.13%
10	26.07	0.84	8.17	-2	3.01%

Table 12: Top 10 Mass Points for Males in the Third Quartile of Pension Savings

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter	Mass
1	344.28	1.46	6.31	-2	13.14%
2	6.07E-05	1.46	10.10	5	12.25%
3	6.07E-05	1.46	13.89	0	11.03%
4	90.66	1.46	8.17	5	8.44%
5	6.07E-05	1.46	8.17	2	8.19%
6	6.07E-05	1.46	10.10	-2	6.64%
7	6.07E-05	1.46	8.17	0	5.93%
8	6.07E-05	1.46	8.17	-2	4.78%
9	0.85	5.00	10.10	0	4.15%
10	90.66	1.46	10.10	10	3.42%

Table 13: Top 10 Mass Points for Males in the Fourth Quartile of Pension Savings

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	-0.04	0.22	-0.32
Risk Aversion	-0.04	1.00	-0.34	-0.27
Outside Wealth	0.22	-0.34	1.00	0.20
Health Shifter	-0.32	-0.27	0.20	1.00

Table 14: Correlation between unobservable types, Female First Quartile

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	-0.22	0.31	-0.15
Risk Aversion	-0.22	1.00	-0.30	-0.08
Outside Wealth	0.31	-0.30	1.00	0.20
Health Shifter	-0.15	-0.08	0.20	1.00

Table 15: Correlation between unobservable types, Female Second Quartile

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	-0.26	0.33	-0.32
Risk Aversion	-0.26	1.00	-0.21	0.14
Outside Wealth	0.33	-0.21	1.00	0.10
Health Shifter	-0.32	0.14	0.10	1.00

Table 16: Correlation between unobservable types, Female Third Quartile

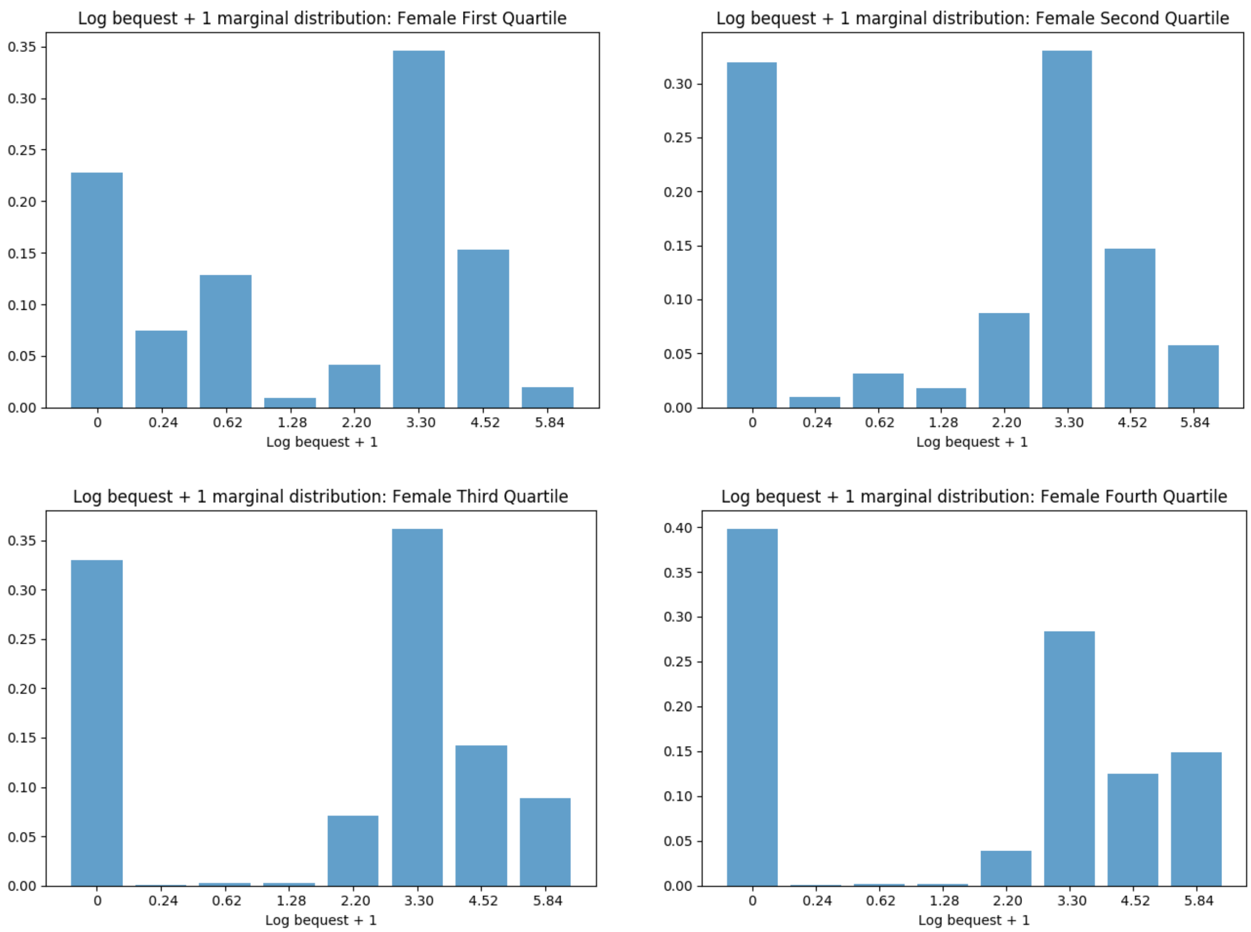


Figure 14: Marginal Distribution of Bequest Motive - Females

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	-0.26	-0.15	-0.36
Risk Aversion	-0.26	1.00	-0.44	0.18
Outside Wealth	-0.15	-0.44	1.00	0.10
Health Shifter	-0.36	0.18	0.10	1.00

Table 17: Correlation between unobservable types, Female Fourth Quartile

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	0.12	-0.01	-0.23
Risk Aversion	0.12	1.00	-0.32	-0.31
Outside Wealth	-0.01	-0.32	1.00	0.23
Health Shifter	-0.23	-0.31	0.23	1.00

Table 18: Correlation between unobservable types, Male First Quartile

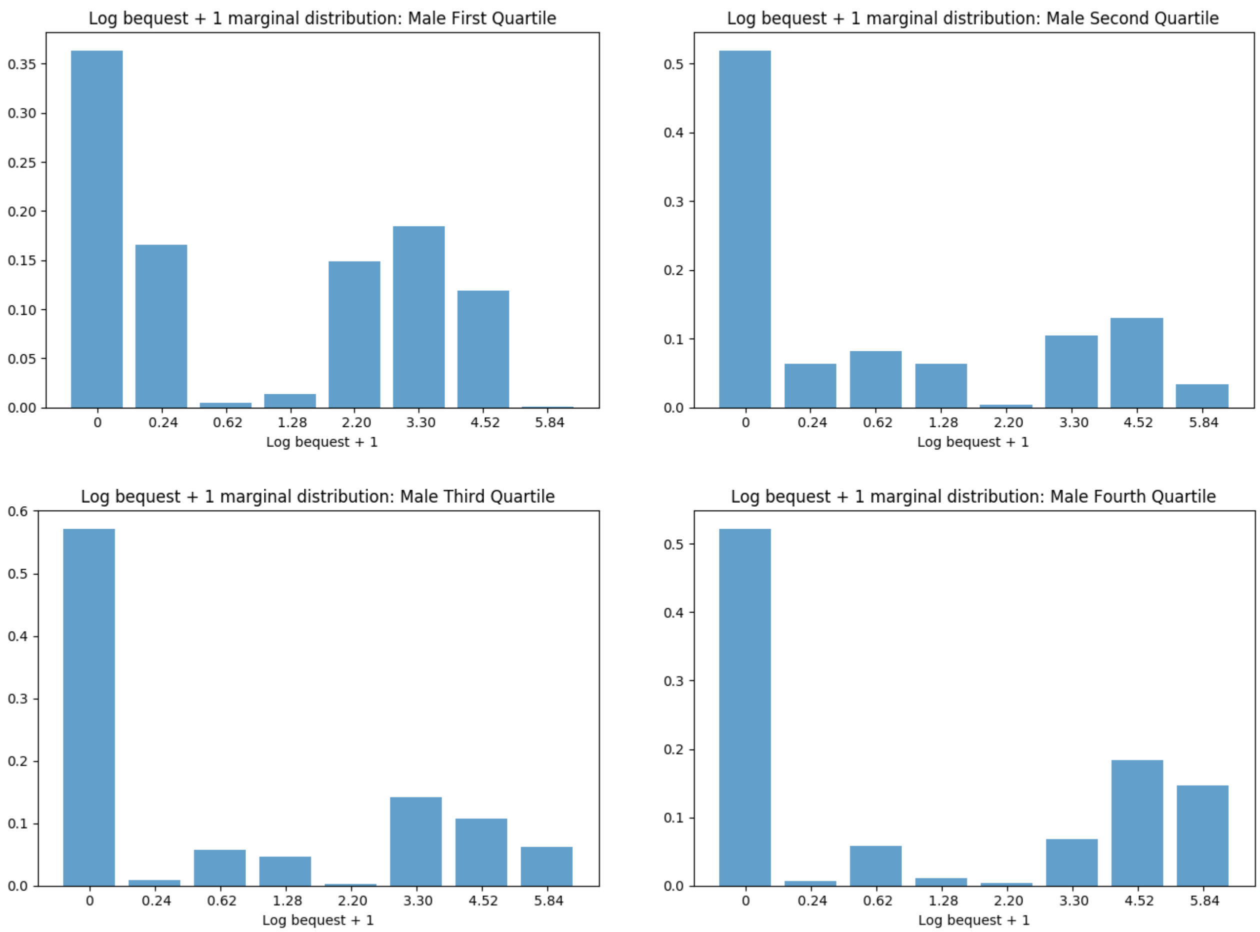


Figure 15: Marginal Distribution of Bequest Motive - Males

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	-0.22	0.44	0.12
Risk Aversion	-0.22	1.00	-0.32	-0.20
Outside Wealth	0.44	-0.32	1.00	0.40
Health Shifter	0.12	-0.20	0.40	1.00

Table 19: Correlation between unobservable types, Male Second Quartile

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	-0.21	0.19	0.10
Risk Aversion	-0.21	1.00	-0.44	-0.12
Outside Wealth	0.19	-0.44	1.00	0.10
Health Shifter	0.10	-0.12	0.10	1.00

Table 20: Correlation between unobservable types, Male Third Quartile

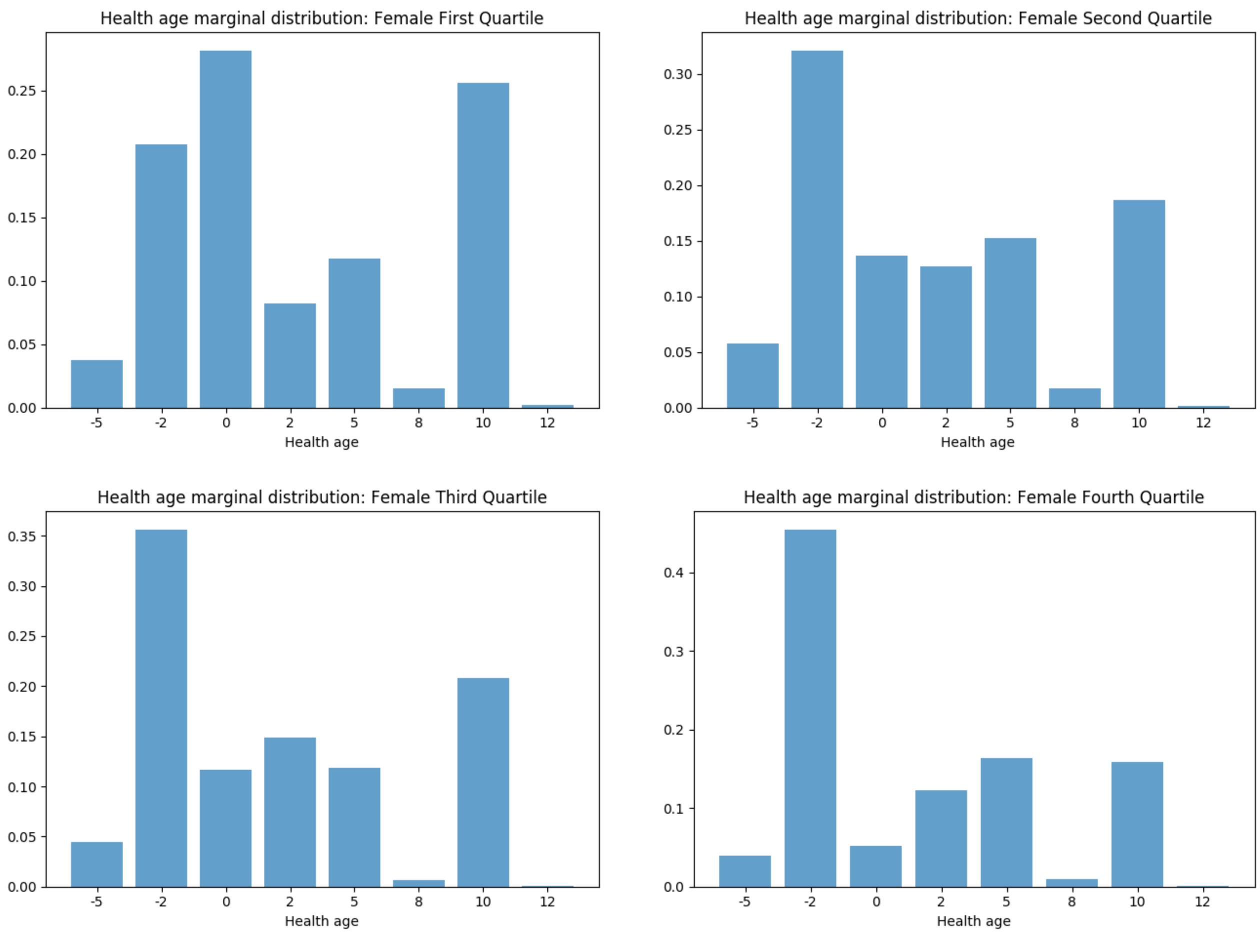


Figure 16: Marginal Distribution of Health Shifter - Females

	Bequest Motive	Risk Aversion	Outside Wealth	Health Shifter
Bequest Motive	1.00	-0.23	0.11	-0.09
Risk Aversion	-0.23	1.00	-0.50	-0.14
Outside Wealth	0.11	-0.50	1.00	0.13
Health Shifter	-0.09	-0.14	0.13	1.00

Table 21: Correlation between unobservable types, Male Fourth Quartile

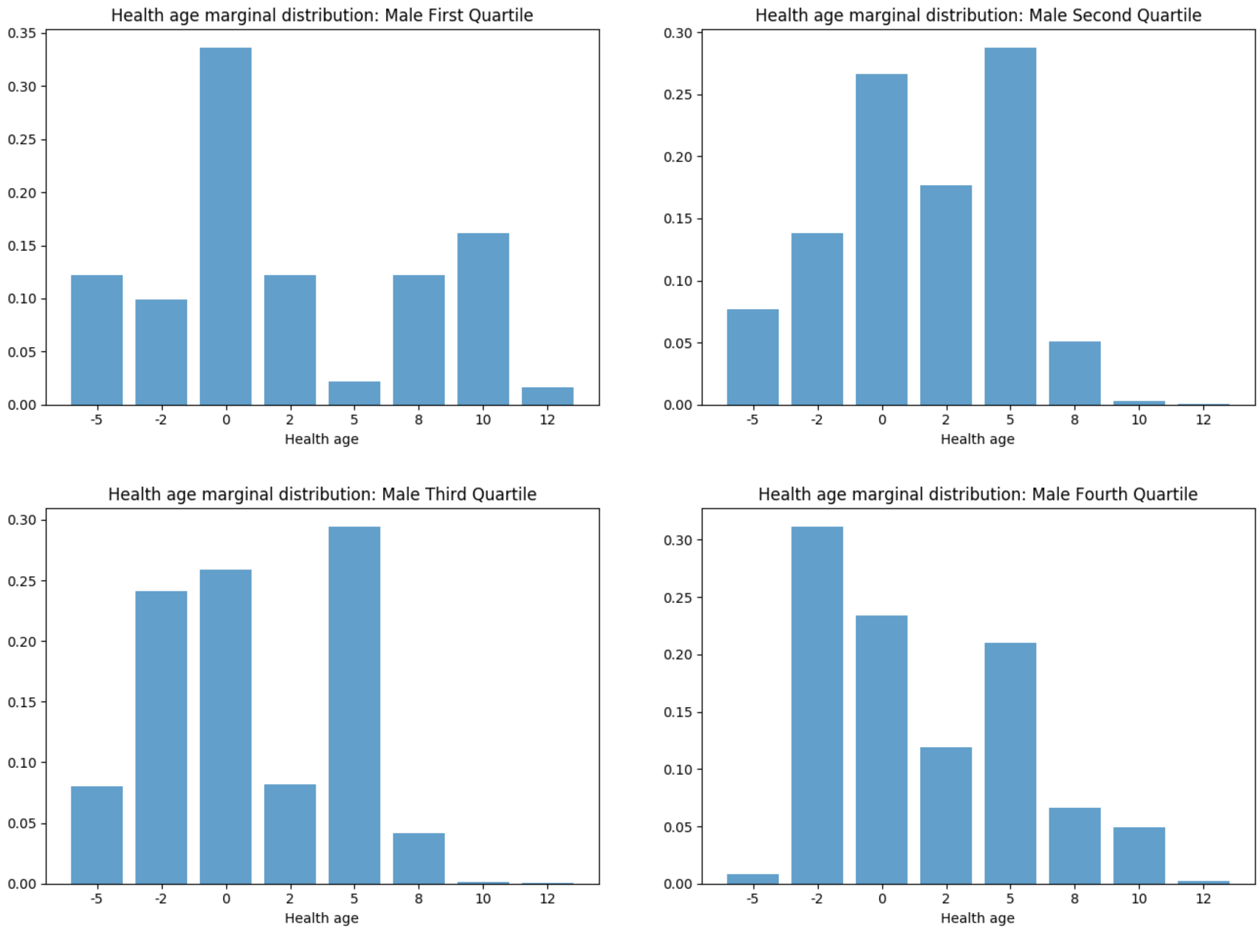


Figure 17: Marginal Distribution of Health Shifter - Males

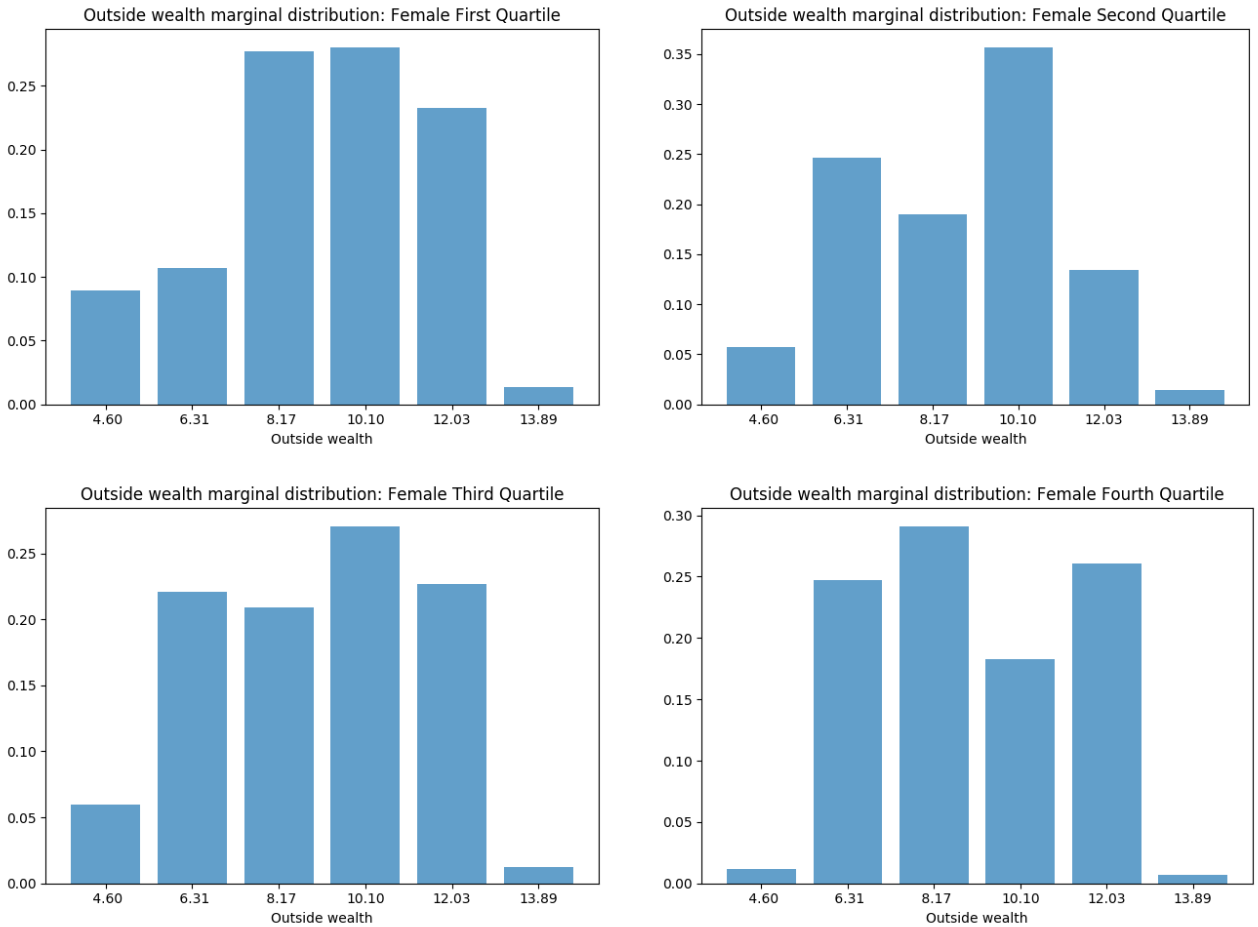


Figure 18: Marginal Distribution of Outside Wealth - Females

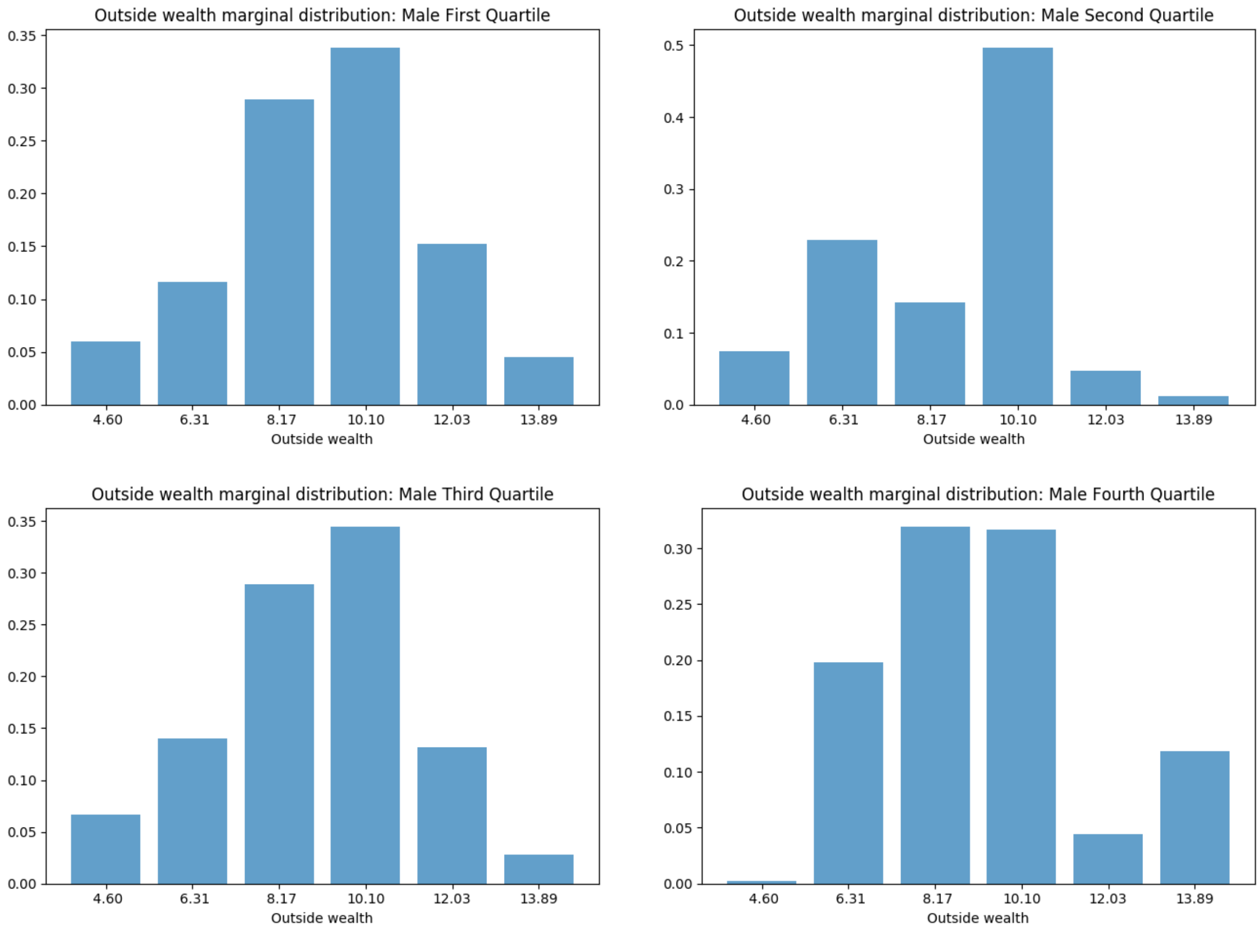


Figure 19: Marginal Distribution of Outside Wealth - Males

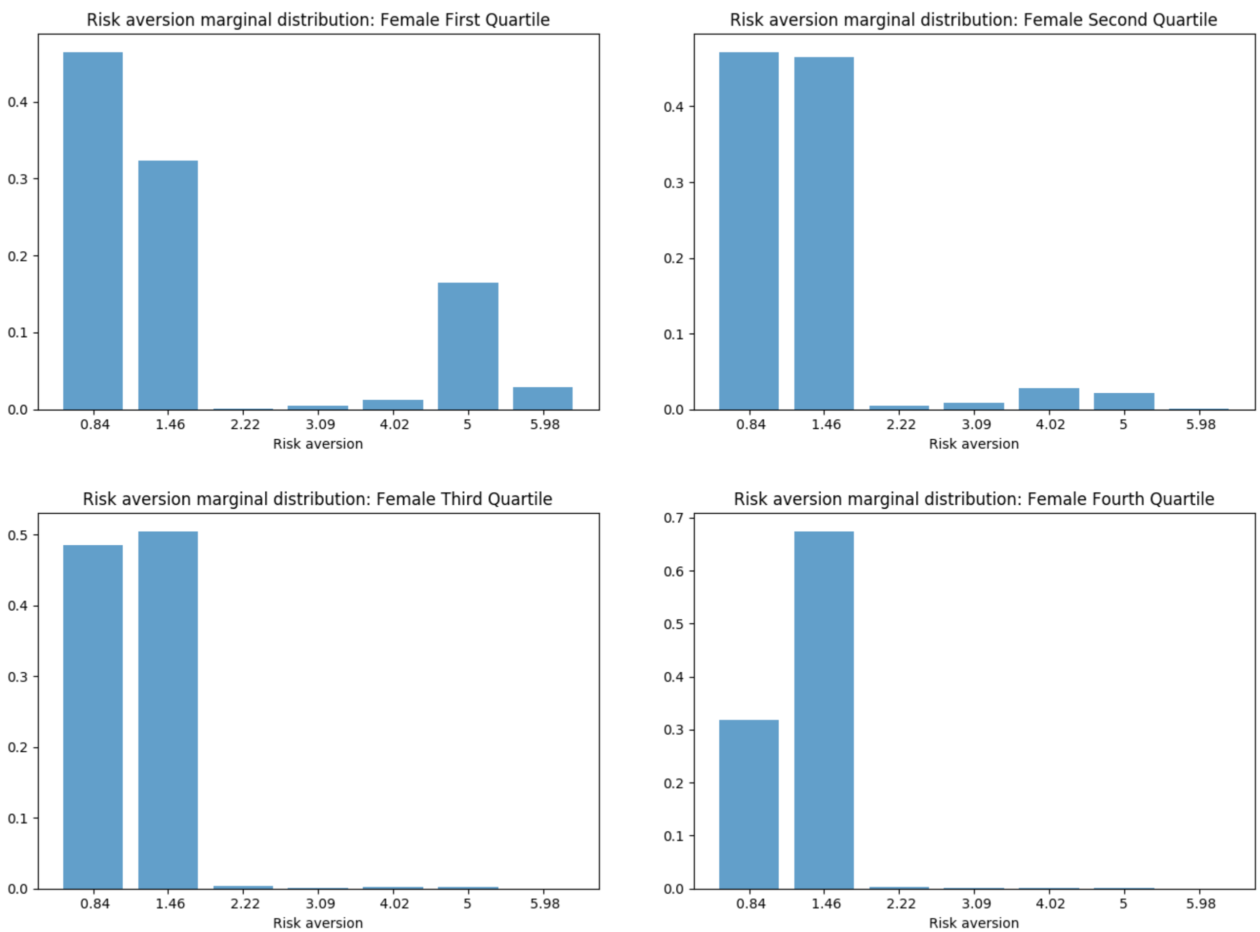


Figure 20: Marginal Distribution of Risk Aversion - Females

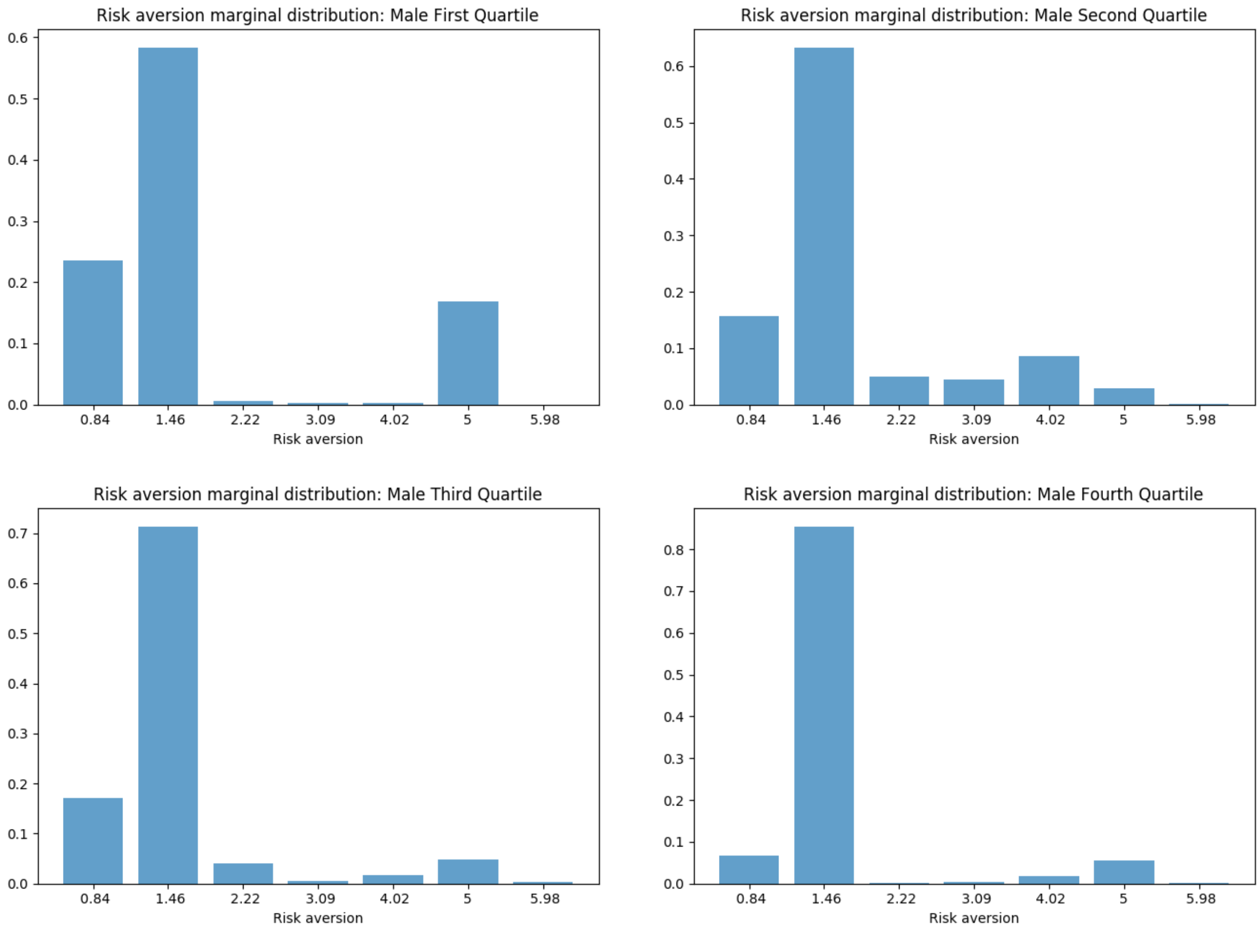
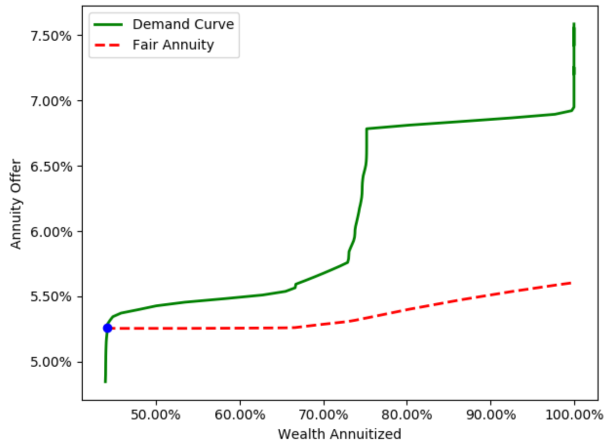


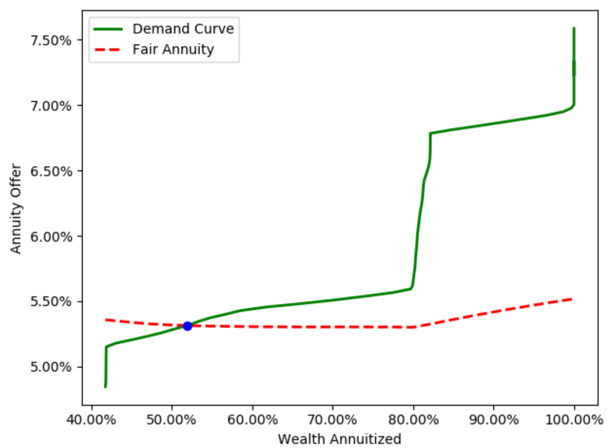
Figure 21: Marginal Distribution of Risk Aversion - Males

Gender	Female				Male			
Wealth Quartile	First	Second	Third	Fourth	First	Second	Third	Fourth
Fraction Annuitized								
Observed	0.65	0.70	0.71	0.64	0.39	0.66	0.71	0.62
Predicted	0.32	0.46	0.52	0.54	0.18	0.36	0.48	0.50
Fraction in Mixed Annuities								
Observed	0.08	0.07	0.07	0.04	0.01	0.02	0.02	0.03
Predicted	0.05	0.05	0.05	0.05	0.02	0.04	0.04	0.07
Fraction in Deferred Annuities								
Observed	0.22	0.30	0.34	0.26	0.06	0.17	0.17	0.15
Predicted	0.07	0.11	0.14	0.13	0.02	0.04	0.06	0.06
Fraction in Guaranteed Annuities								
Observed	0.53	0.59	0.60	0.52	0.25	0.48	0.48	0.41
Predicted	0.12	0.23	0.27	0.26	0.03	0.09	0.16	0.19
Two-year mortality								
Observed	1.55%	1.71%	1.32%	1.33%	6.39%	5.42%	4.37%	2.95%
Predicted	1.23%	1.14%	1.14%	1.15%	3.77%	3.05%	2.79%	2.98%
Number of observations	426566	692103	738509	697265	65402	139733	181948	210611
Number of consumers	9083	9180	9023	8412	2768	2800	2735	2676
Unobserved heterogeneity levels	194	194	194	194	194	194	194	194
MSE	0.02	0.01	0.01	0.01	0.03	0.02	0.02	0.01
R2	0.60	0.47	0.43	0.42	0.74	0.58	0.48	0.45

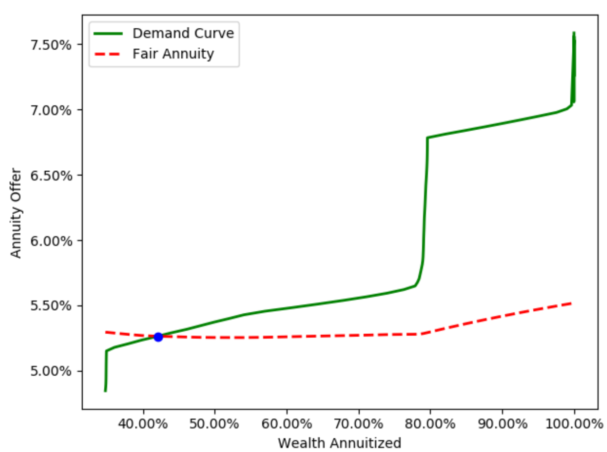
Table 22: Various measures of in and out-of-sample fit



(a) First Quartile

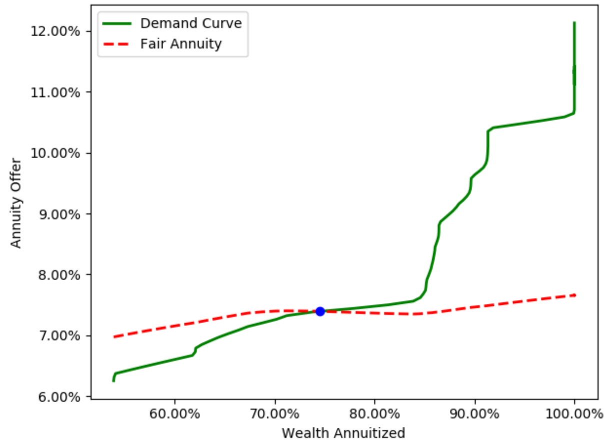


(b) Second Quartile

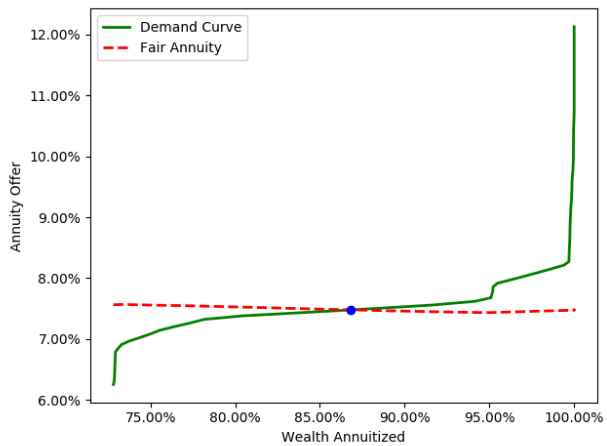


(c) Third Quartile

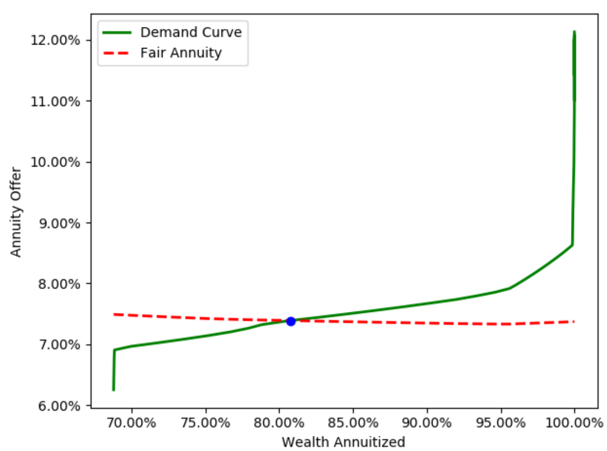
Figure 22: Simulated Equilibria under Chilean System - Females



(a) First Quartile

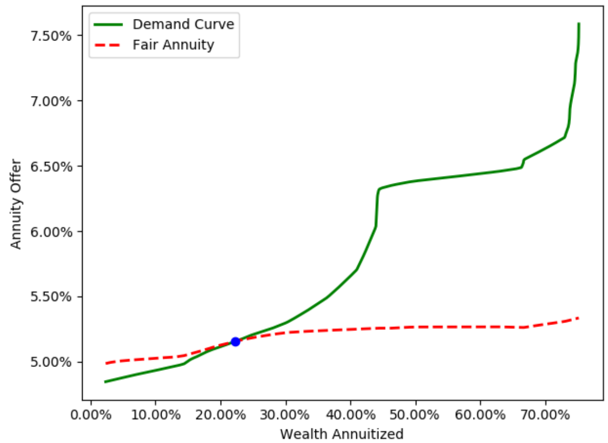


(b) Second Quartile

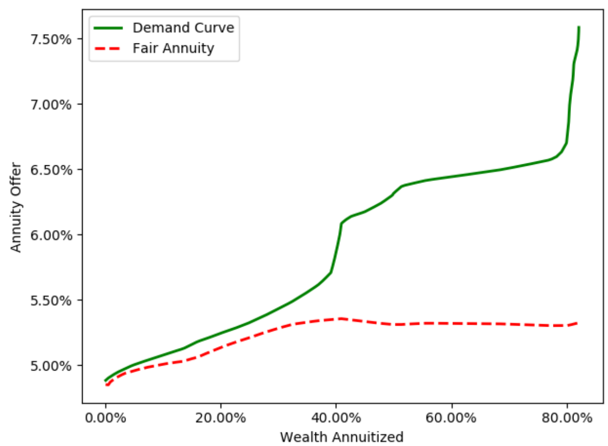


(c) Third Quartile

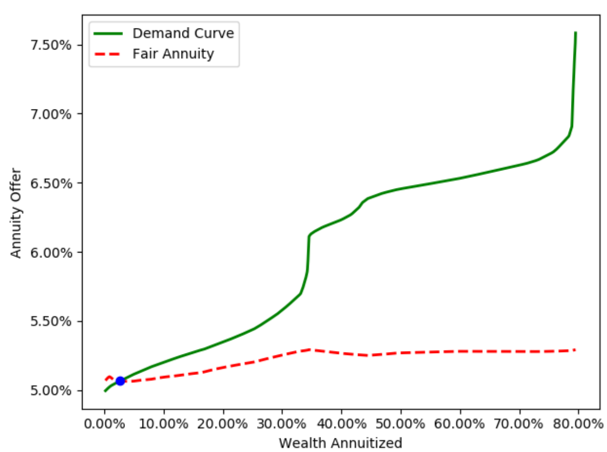
Figure 23: Simulated Equilibria under Chilean System - Males



(a) First Quartile

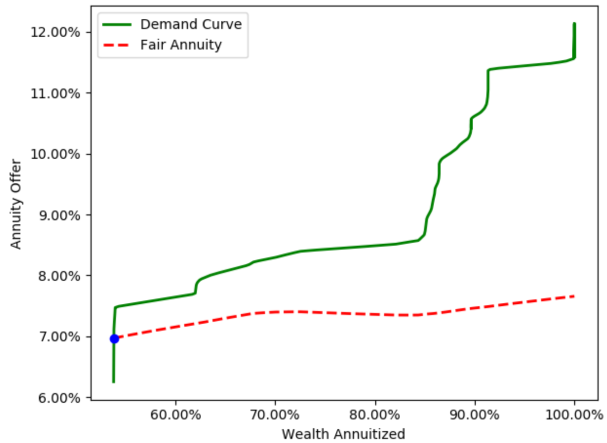


(b) Second Quartile

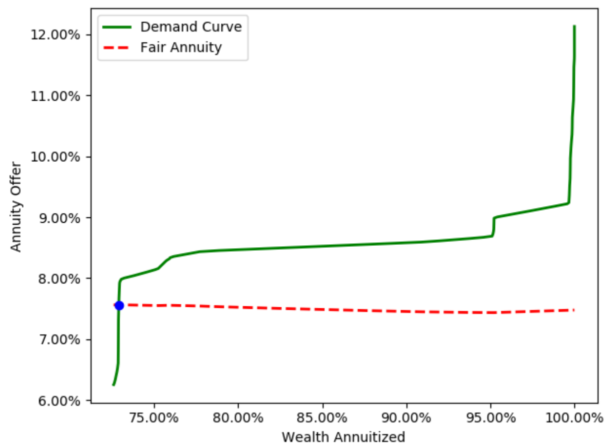


(c) Third Quartile

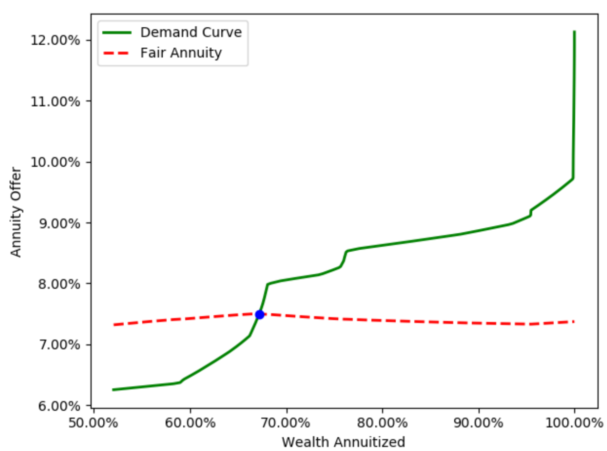
Figure 24: Simulated Equilibria under US System - Females



(a) First Quartile

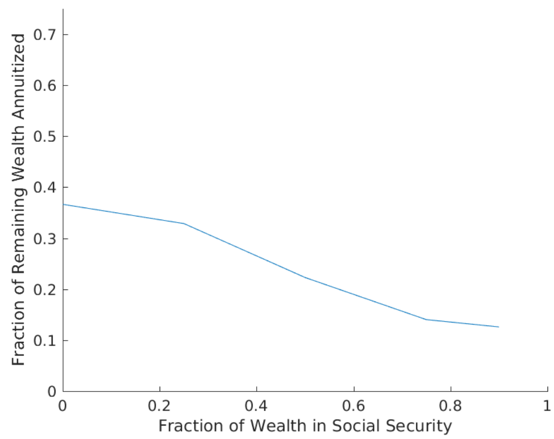


(b) Second Quartile

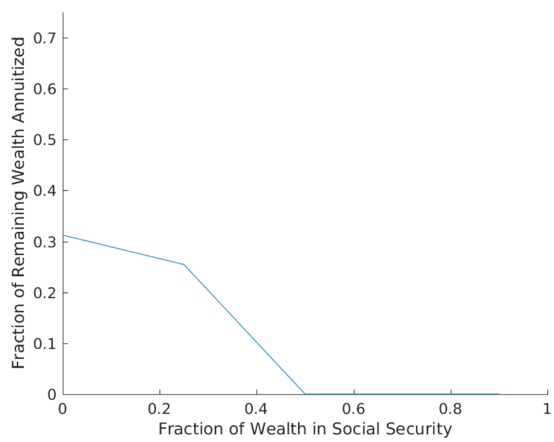


(c) Third Quartile

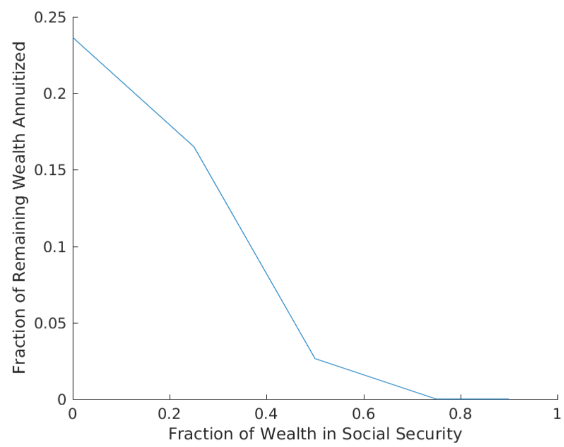
Figure 25: Simulated Equilibria under US System - Males



(a) First Quartile

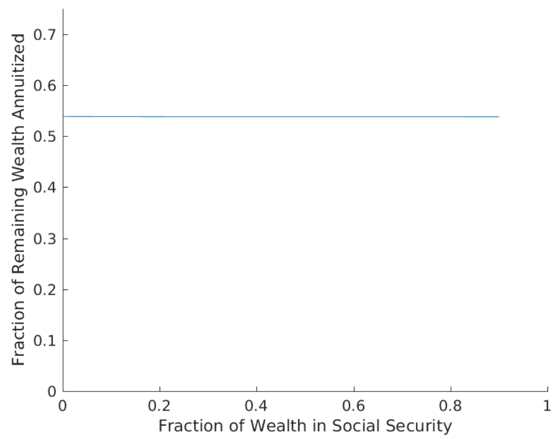


(b) Second Quartile

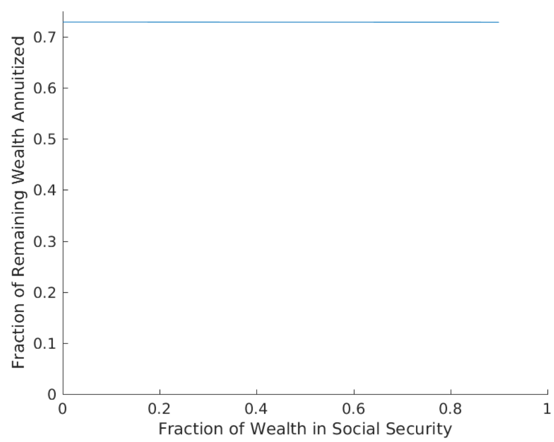


(c) Third Quartile

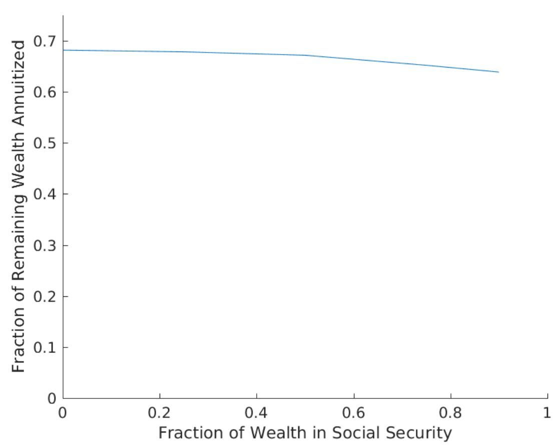
Figure 26: Simulated equilibria under US system for different levels of wealth in Social Security - Females



(a) First Quartile



(b) Second Quartile



(c) Third Quartile

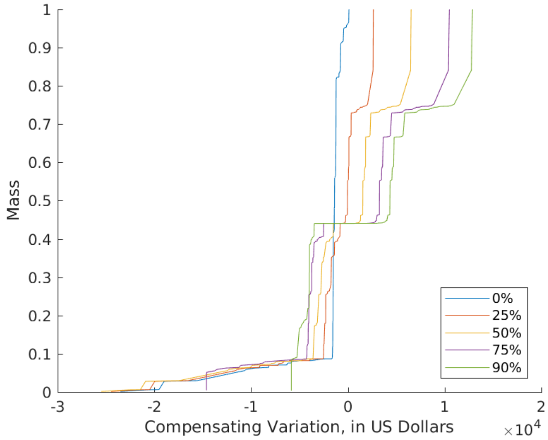
Figure 27: Simulated equilibria under US system for different levels of wealth in Social Security - Males

Annuity Payouts under the Chilean and US Equilibria						
	Panel A: Equilibrium Annuity Amount					
	Female Q1	Female Q2	Female Q3	Male Q1	Male Q2	Male Q3
Actuarially Fair	5.61%	5.52%	5.52%	7.66%	7.48%	7.37%
Chilean Equilibrium	5.26%	5.31%	5.26%	7.39%	7.48%	7.39%
US Equilibrium, 0% SS	5.24%	5.29%	5.23%	6.97%	7.57%	7.49%
US Equilibrium, 25% SS	5.23%	5.23%	5.18%	6.97%	7.56%	7.49%
US Equilibrium, 50% SS	5.16%	-	5.06%	6.97%	7.56%	7.50%
US Equilibrium, 75% SS	5.04%	-	-	6.97%	7.56%	7.50%
US Equilibrium, 90% SS	5.03%	-	-	6.97%	7.56%	7.51%
	Panel B: Full Annuitization Annuity Payout					
	Female Q1	Female Q2	Female Q3	Male Q1	Male Q2	Male Q3
Chilean Equilibrium	5.26%	5.31%	5.26%	7.39%	7.48%	7.39%
US Equilibrium, 0% SS	5.24%	5.29%	5.23%	6.97%	7.57%	7.49%
US Equilibrium, 25% SS	5.32%	5.30%	5.26%	7.14%	7.54%	7.46%
US Equilibrium, 50% SS	5.38%	-	5.29%	7.32%	7.52%	7.44%
US Equilibrium, 75% SS	5.46%	-	-	7.49%	7.50%	7.41%
US Equilibrium, 90% SS	5.55%	-	-	7.59%	7.49%	7.39%

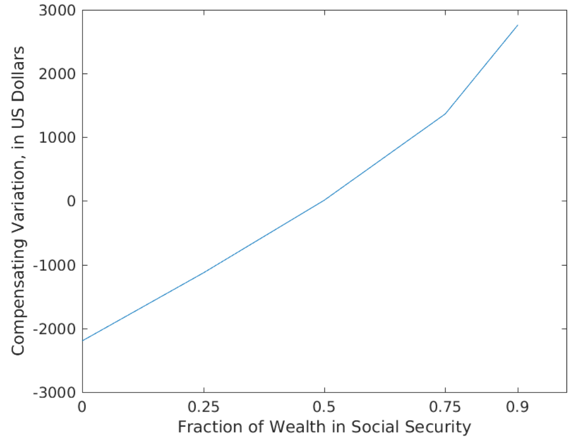
Table 23: Comparison of annuity payouts under the Chilean and US Equilibria

Choices in the Chilean Equilibrium			
	Full Annuitization	Full PW	Mixed
Female Q1	44.05%	55.75%	0.20%
Female Q2	45.11%	40.11%	14.78%
Female Q3	34.89%	49.42%	15.68%
Male Q1	68.51%	15.66%	15.83%
Male Q2	75.90%	5.01%	19.10%
Male Q3	76.21%	4.59%	19.20%

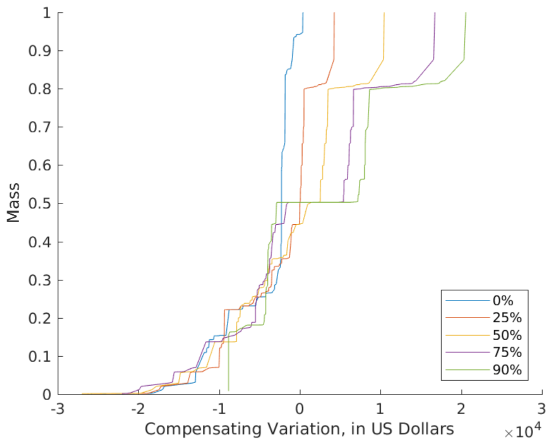
Table 24: Allocation of types to choices in the Chilean equilibrium



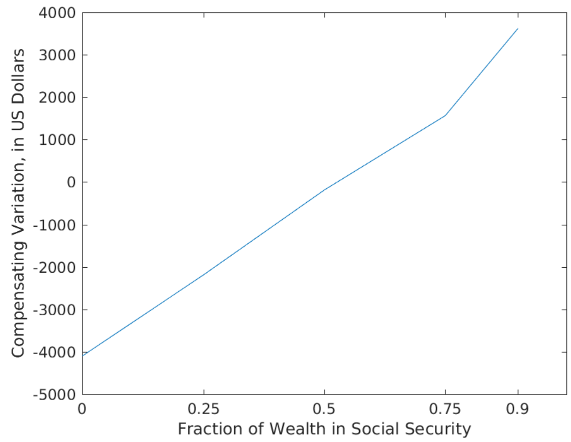
(a) First Quartile CDFs



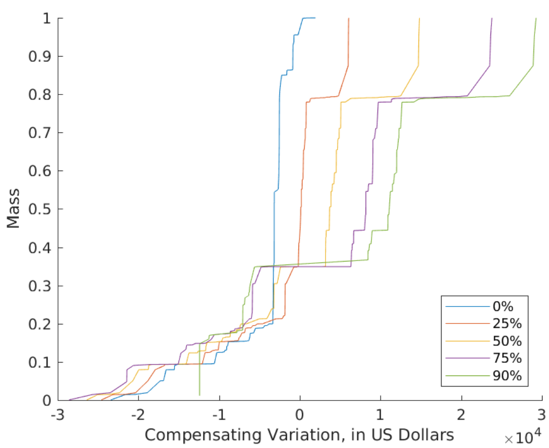
(b) First Quartile Means



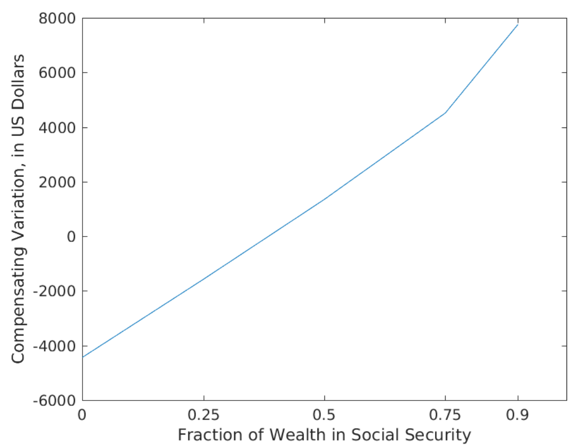
(c) Second Quartile CDFs



(d) Second Quartile Means

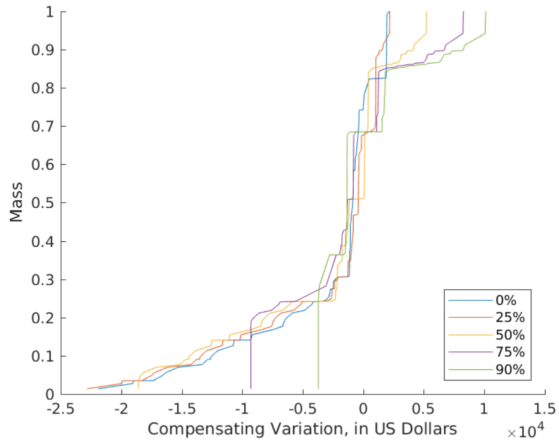


(e) Third Quartile CDFs

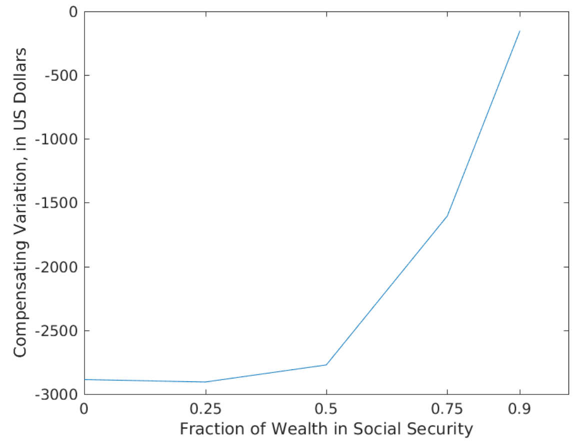


(f) Third Quartile Means

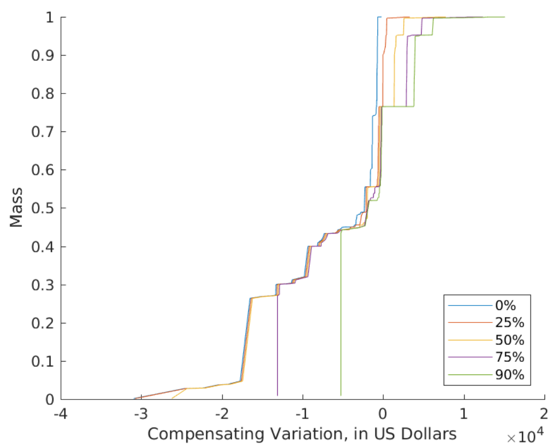
Figure 28: Female compensating variation CDFs and means, for different fractions of wealth in Social Security



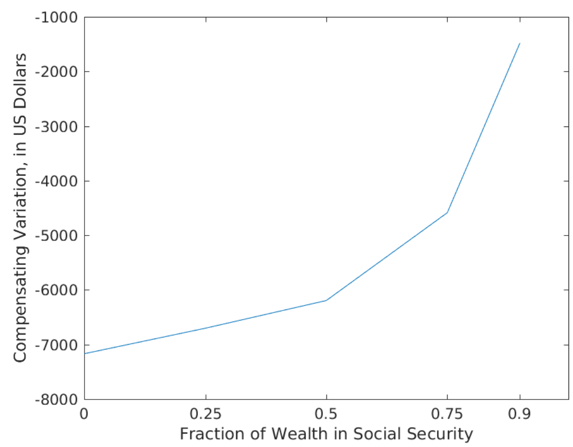
(a) First Quartile



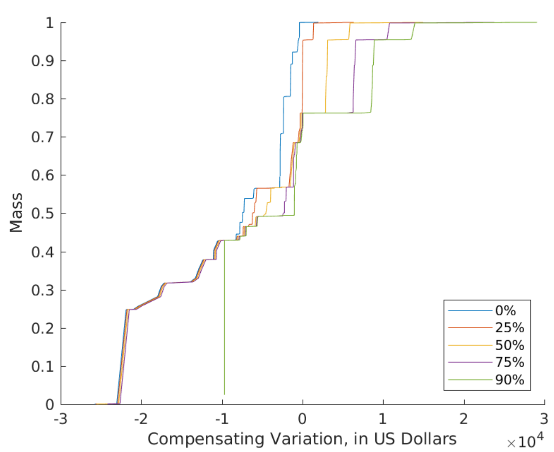
(b) First Quartile Means



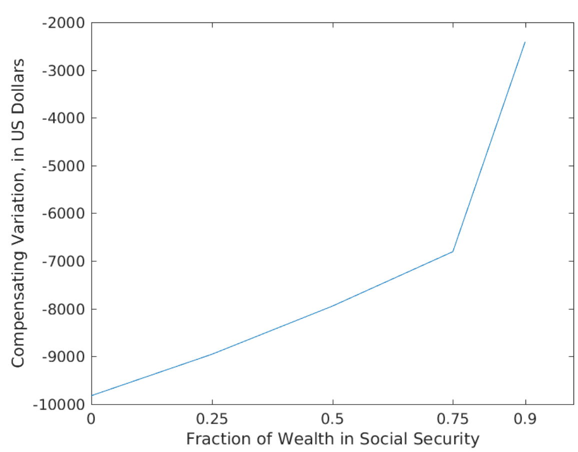
(c) Second Quartile



(d) Second Quartile Means



(e) Third Quartile



(f) Third Quartile Means

Figure 29: Male compensating variation CDF - the amount of additional pension balances that leave a type indifferent between the US system and the Chilean system - for different fractions of wealth in Social Security

B Regulatory Details

This appendix presents the formulas for programmed withdrawal and the minimum pension guarantee in detail.

B.1 Programmed Withdrawal

The exposition in this subsection follows Pino (2005). PW payouts in each month of year t for an individual of age x and gender g are

$$PW_t(x,g) = \frac{Balance_t}{CNU_t(x,g) \cdot 12} \tag{20}$$

where balance is the beginning of year account balance in the PFA and CNU is the expected present discounted value of paying out a unit pension. To calculate the CNU, we need to define a few objects. A mortality table issued in year m defines a gender-specific death probability $q^m(x,g)$ for every age x and an adjustment factor $AF^m(x,g)$ - a value meant to correct for increasing longevity expectations for a fixed mortality table.

In year t, the appropriate value for $q^t(x,g)$ is

$$q^t(x,g) = q^m(x,g) \cdot *(1 - AF^m(x,g))^{t-m} \tag{21}$$

Regardless of gender, the tables assume that the probability of being alive at 20 equals 1 and that the probability of being alive at 110 equals 0. For intermediate values, define $l^t(x,g)$, the year t probability of being alive at age x, as

$$l^t(x,g) = l^t(x-1,g) \cdot (1 - q^t(x-1,g)) \text{ for } x \in (20,110] \tag{22}$$

Then $CNU_t(x,g)$ is

$$CNU_t(x,g) = \sum_{j=x}^{110} \frac{l^t(j,g)}{l^t(x,g) \cdot (1+r_{RP})^{j-x}} - \frac{11}{24} \text{ for } x \in (20,110] \tag{23}$$

where $r_{RP} = 0.8 \cdot r_A + 0.2 \cdot \bar{r}$, r_A is the previous year's implicit interest rate for annuities and $\bar{r}$ is the 10 year average return for PW balances. Finally, note that CNU calculations vary for individuals with dependents. We do not report those adjustments, as we work with a no-dependents sample. See Pino (2005) for details. Readers wishing to obtain CNU values will benefit from also reading Vega (2014) and the accompanying Stata module.

B.2 Minimum Pension Guarantee

There are two minimum pension regimes in Chile during our sample period: pre and post 2008. In the first period, any individual with at least 20 years of contributions into the pension system who receives a pension below a minimum guaranteed amount receives a top-up from the government. Since annuity offers cannot fall below this amount, during this period the minimum guaranteed amount is only relevant for valuing programmed withdrawal contracts and for calculating annuity payouts after a default. We value both contracts by taking the UF denominated value of the pension guarantee at the time of retirement and holding it fixed throughout the lifetime of the contract.

Starting in 2008, this guarantee is replaced by an expanded top-up that is available to individuals whose pension falls below a maximum amount. To be precise, the new regime sets a new floor, called the "Pensión Básica Solidaria" or PBS, and a maximum, called the "Pensión Máxima con Aporte Solidario", or PMAS. Annuity offers after this reform cannot fall below the PBS, and individuals funding offers above the PMAS receive no subsidy. For individuals who fund an offer ("Pensión Base", or PB) in between the PBS and the PMAS, the government top up ("Complemento Solidario", or CS) is

$$CS = PBS \cdot (1 - \frac{PB}{PMAS}) \tag{24}$$

This amount is added to any annuity offer accepted, regardless of contract type, provided the retiree is 65 or older, has lived in Chile for 20 years after the age of 20, has lived in Chile for 4 of the last 5 years, and is in the 60% percentile or lower in a needs-based poverty index ("Puntaje de Focalización Previsional").

For PW offers, a corrected version of the CS is added to the payout schedule. The correction is meant to ensure that the expected present discounted value of the subsidy is equal under PW and an annuity. See Com (2018) for details.

C Model

This appendix section presents the detailed explanation of how the values of annuity and programmed withdrawal offers are calculated. It is divided into four subsections. The first derives the Euler equations for the annuity problem; the second derives the Euler equations for the PW problem; the third presents the computational details of how to solve the annuity problem; and the fourth does the same for the PW problem.

C.1 Derivations for the Annuity Problem

Consider the problem presented in Equation 4. For expositional clarity, we ignore the no borrowing constraint and derive a solution in an unconstrained setting, and then bring the constraint back in. It is well known that the problems of the previous form can be re-written recursively. In any arbitrary period t, the value of the remaining consumption problem given the current death state d_t, bankruptcy state b_t and liquid assets m_t is $V_t(d_t, q_t, m_t)$, and the Bellman equations are:

$$V_t(d_t, q_t, m_t) = \max_{c_t(d_t, q_t)} \frac{c_t(d_t, q_t)^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_t(d_t, q_t)' \begin{bmatrix} E_t\left[V_{t+1}(0,0,m_{t+1})\right] \\ E_t\left[V_{t+1}(0,1,m_{t+1})\right] \\ E_t\left[V_{t+1}(1,0,m_{t+1})\right] \\ E_t\left[V_{t+1}(1,1,m_{t+1})\right] \end{bmatrix} \tag{25}$$

where $\Gamma_t(0,0) = \begin{bmatrix} (1-\mu_{t+1})(1-\psi_{t+1}) \\ (1-\mu_{t+1})\psi_{t+1} \\ \mu_{t+1}(1-\psi_{t+1}) \\ \mu_{t+1}\psi_{t+1} \end{bmatrix}$, $\Gamma_t(0,1) = \begin{bmatrix} 0 \\ (1-\mu_{t+1}) \\ 0 \\ \mu_{t+1} \end{bmatrix}$, $\Gamma_t(1,0) = \begin{bmatrix} 0 \\ 0 \\ (1-\psi_{t+1}) \\ \psi_{t+1} \end{bmatrix}$, and $\Gamma_t(1,1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, and each equation is subject to the appropriate dynamic budget constraints and transition rules. We can simplify the previous equation by noting that there is no optimization after death, so for the absorbing state $(d_t = 1, q_t = 1)$ we have that:

$$\begin{aligned} V_t(1,1,m_t) &= \beta \frac{\left[m_t + PDV_t^z(1,1,D,G)\right]^{1-\gamma}}{1-\gamma} \\ E_t\left[V_{t+1}(1,1,m_{t+1})\right] &= \beta \frac{\left[m_{t+1} + PDV_{t+1}^z(1,1,D,G)\right]^{1-\gamma}}{1-\gamma} \end{aligned} \tag{26}$$

where $PDV_t^z(1,1,D,G) = \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot z_\tau(1,1,D,G)$ is the PDV in period t of the payment stream of the guarantee period from $t+1$ to $G+D$.

The expressions are similar in the "dead but not bankrupt" case $(d_t = 1, q_t = 0)$, but take into account that for guaranteed annuities there is uncertainty in the value of future payments:

$$\begin{aligned} V_t(1,0,m_t) &= \beta \frac{\left[m_t + E[PDV_t^z(1,0,D,G)]\right]^{1-\gamma}}{1-\gamma} \\ E_t\left[V_{t+1}(1,0,m_{t+1})\right] &= \beta \frac{\left[m_{t+1} + E[PDV_{t+1}^z(1,0,D,G)]\right]^{1-\gamma}}{1-\gamma} \end{aligned} \tag{27}$$

where $E[PDV_t^z(1,0,D,G)]$ is the expected present value in t of the payment stream of the guarantee period from $t+1$ to $G+D$:

$$E[PDV_t^z(1,0,D,G)] = \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot \left((1-\Psi_\tau) \cdot z_\tau(1,0,D,G) + \Psi_\tau \cdot z_\tau(1,1,D,G)\right) \tag{28}$$

$$\Psi_\tau = \sum_{\kappa=t+1}^{\tau} \left(\prod_{\tilde{\kappa}=t+1}^{\kappa-1} (1-\psi_{\tilde{\kappa}}) \right) \psi_\kappa \tag{29}$$

and Ψ_τ is the probability that the firm is bankrupt in $\tau > t$, conditional on not being bankrupt in t.
As for the remaining states (when the individual is alive), the FOCs from (25) are:

$$c_t(0,q_t)^{-\gamma} = \delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} E_t\left[V'_{t+1}(0,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(0,1,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,1,m_{t+1})\right] \end{bmatrix} \tag{30}$$

We know that:

$$\begin{aligned} E_t\left[V'_{t+1}(1,0,m_{t+1})\right] &= \beta \cdot [m_{t+1} + \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot ((1-\Psi_\tau)\cdot z_\tau(1,0,D,G) + \Psi_\tau \cdot z_\tau(1,1,D,G))]^{-\gamma} \\ E_t\left[V'_{t+1}(1,1,m_{t+1})\right] &= \beta \cdot \left[m_{t+1} + \sum_{\tau=t+1}^{G+D} R^{t-\tau} \cdot z_\tau(1,1,D,G)\right]^{-\gamma} \end{aligned} \tag{31}$$

Also, from the Envelope Theorem:

$$V'_t(0,q_t,m_t) = \delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} E_t\left[V'_{t+1}(0,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(0,1,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,1,m_{t+1})\right] \end{bmatrix} \tag{32}$$

Combining (30) and (32), and rolling the equation forward by one year:

$$\begin{aligned} c_t(0,q_t)^{-\gamma} &= V'_t(0,q_t,m_t) \\ c_{t+1}(0,q_{t+1})^{-\gamma} &= V'_{t+1}(0,q_{t+1},a_t \cdot R + z_{t+1}(0,q_{t+1},D,G)) \end{aligned} \tag{33}$$

Substituting back into (30) yields the Euler equation:

$$c_t(0,q_t)^{-\gamma} = \delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} E_t\left[c_{t+1}(0,0)^{-\gamma}\right] \\ E_t\left[c_{t+1}(0,1)^{-\gamma}\right] \\ E_t\left[V'_{t+1}(1,0,m_{t+1})\right] \\ E_t\left[V'_{t+1}(1,1,m_{t+1})\right] \end{bmatrix} \tag{34}$$

Following Carroll (2012), note that in equation (33) neither m_t nor c_t has any direct effect on V'_{t+1}. Instead, it is their difference, a_t, which enters into the function. This motivates the use of the Endogenous Gridpoint Method to approximate the optimal policy and value functions, as is derived in subsection C.3. Before moving to computation, however, the next section presents the analogous derivation for the PW problem.

C.2 Derivations for the PW Problem

Consider for now the problem free of borrowing constraint. As before, utility is CRRA, and in each state is given by:

$$u(c_t, d_t = 0) = \frac{c_t^{1-\gamma}}{1-\gamma}$$
$$u(d_t = 1) = \beta \cdot \frac{(m_t + PW_t)^{1-\gamma}}{1-\gamma} \tag{35}$$

As in the annuity case, to obtain the value of taking a PW offer we re-write the problem in recursive form. The Bellman equation for the PDV of expected utility under the optimal state-contingent consumption path, for any period t, given the death state, PW account balance, and asset balance, denoted by $V_t(d_t, PW_t, m_t)$, is:

$$V_t(d_t = 0, PW_t, m_t) = \max_{c_t} \frac{c_t^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_t' \begin{bmatrix} E_t\left[V_{t+1}(0, m_{t+1}, PW_t)\right] \\ E_t\left[V_{t+1}(1, m_{t+1}, PW_t)\right] \end{bmatrix} \tag{36}$$

where $\Gamma_t = \begin{bmatrix} 1-\mu_{t+1} \\ \mu_{t+1} \end{bmatrix}$ and, as before, the problem is constrained by dynamic budget constraints and transition rules. Since there is no optimization after death, and inheritors receive the full PW balance, for the absorbing state $d_t = 1$ we have that:

$$V_t(1, m_t, PW_t) = \beta \frac{\left[m_t + PW_t\right]^{1-\gamma}}{1-\gamma} \tag{37}$$

Therefore we can write the expected continuation value for the death state as:

$$E_t[V_{t+1}(1, m_{t+1}, PW_t)] = \frac{\beta}{1-\gamma} \int \left[m_{t+1} + (PW_t - z_t(PW_t)) \cdot R^{PW}\right]^{1-\gamma} dF(R^{PW}) \tag{38}$$

For the state where the individual is alive, the expected continuation value is:

$$E_t[V_{t+1}(0, m_{t+1}, PW_t)] = \int V_{t+1}(0, (PW_t - z_t(PW_t)) \cdot R^{PW}, m_{t+1}) dF(R^{PW}) \tag{39}$$

With these definitions, the FOCs from (36) are:

$$c_t^{-\gamma} = \delta \cdot R \cdot \Gamma_t' \begin{bmatrix} E_t\left[V_{t+1}'(0, m_{t+1}, PW_t)\right] \\ E_t\left[V_{t+1}'(1, m_{t+1}, PW_t)\right] \end{bmatrix} \tag{40}$$

We know that:

$$E_t\left[V_{t+1}'(1, m_{t+1}, PW_t)\right] = \beta \cdot R \int \left[m_{t+1} + (PW_t - z_t(PW_t)) \cdot R^{PW}\right]^{-\gamma} dF(R^{PW}) \tag{41}$$

Also, from the Envelope Theorem:

$$V_t'(0,m_t) = \delta \cdot R \cdot \Gamma_t' \begin{bmatrix} E_t\left[V_{t+1}'(0,m_{t+1},PW_t)\right] \\ E_t\left[V_{t+1}'(1,m_{t+1},PW_t)\right] \end{bmatrix} \tag{42}$$

Combining (40) and (42), and rolling the equation forward by one year:

$$\begin{aligned} c_t^{-\gamma} &= V_t'(0,m_t) \\ c_{t+1}^{-\gamma} &= V_{t+1}'(0,m_{t+1},PW_{t+1}) \end{aligned} \tag{43}$$

Substituting back into (40) yields the Euler equation:

$$c_t^{-\gamma} = \delta \cdot R \cdot \Gamma_t' \begin{bmatrix} E_t\left[c_{t+1}^{-\gamma}\right] \\ E_t\left[V_{t+1}'(1,m_{t+1},PW_t)\right] \end{bmatrix} \tag{44}$$

C.3 Computation of the Solution to the Annuity Problem

Having derived the conditions that govern the optimal consumption policy and the value functions for both problems, this subsection presents the details of the numerical procedure used to solve these conditions. Since the problem is solved recursively, we will begin with the solution for period T and work our way backwards. In period T, $\mu_T = 1$ and $T > G+D$, so $m_T = a_{T-1} \cdot R$ and regardless of the bankruptcy state q_T:

$$V_T(0,q_T,m_T) = \beta \cdot \frac{m_T^{1-\gamma}}{1-\gamma} \tag{45}$$

Then in the next-to-last period:

$$V_{T-1}(0,q_{T-1},m_{T-1}) = \max_{c_{T-1}} \frac{c_{T-1}^{1-\gamma}}{1-\gamma} + \delta \cdot \beta \cdot \frac{((m_{T-1}-c_{T-1}) \cdot R)^{1-\gamma}}{1-\gamma} \tag{46}$$

Which generates the optimal policy:

$$\begin{aligned} c_{T-1}^{-\gamma} &= \delta \cdot \beta \cdot R^{1-\gamma} \cdot (m_{T-1}-c_{T-1})^{-\gamma} \\ c_{T-1}(0,q_{T-1},m_{T-1}) &= \frac{R}{((\delta \cdot \beta \cdot R)^{\frac{1}{\gamma}} + R)} \cdot m_{T-1} \end{aligned} \tag{47}$$

And implies that the value function in $T-1$ is:

$$V_{T-1}(0,q_{T-1},m_{T-1}) = \left(\frac{1+(\delta \cdot \beta \cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma}\right)\left(\frac{R \cdot m_{T-1}}{(\delta \cdot \beta \cdot R)^{1/\gamma}+R}\right)^{1-\gamma} \tag{48}$$

Note that conditional on m_{T-1}, there is no dependence on q_{T-1}. That is, q_{T-1} will shift m_{T-1}, as $m_{T-1} = a_{T-2} \cdot R + z_{T-1}(0, q_{T-1}, D, G)$, but conditional on m_{T-1} it becomes irrelevant. Therefore, given a grid of m_{T-1} one could easily solve for $V_{T-1}(m_{T-1})$, and the value of m_{T-1}'s for other values would be found by interpolation/extrapolation. Note as well that as long as the bequest motive is positive the no-borrowing constraint can be omitted from this stage without loss as the the unconstrained solution always satisfies $c_{T-1} < m_{T-1}$.

Having solved for all the relevant quantities in $T-1$ and T, let us consider the unconstrained problem in $T-2$. From the Euler condition in (34) and the optimal policy in (47):

$$c_{T-2}(0,q_t)^{-\gamma} = \delta \cdot R \cdot \Gamma_{T-2}(0,q_t)' \begin{bmatrix} E_t\left[c_{T-1}(0,0)^{-\gamma}\right] \\ E_t\left[c_{T-1}(0,1)^{-\gamma}\right] \\ E_t\left[V'_{T-1}(1,0,m_{T-1})\right] \\ E_t\left[V'_{T-1}(1,1,m_{T-1})\right] \end{bmatrix}$$

$$= \delta \cdot R \cdot \Gamma_{T-2}(0,q_t)' \begin{bmatrix} \left(\frac{R}{((\delta\cdot\beta\cdot R)^{1/\gamma}+R)}\right)^{-\gamma} ((m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R + z_{T-1}(0,0,D,G))^{-\gamma} \\ \left(\frac{R}{((\delta\cdot\beta\cdot R)^{1/\gamma}+R)}\right)^{-\gamma} ((m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R + z_{T-1}(0,1,D,G))^{-\gamma} \\ \beta\cdot\left[(m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R + z_{T-1}(1,0,D,G) + E[PDV^z_{T-1}(1,0,D,G)]\right]^{-\gamma} \\ \\ \beta\cdot\left[(m_{T-2}-c_{T-2}(0,q_{T-2}))\cdot R + z_{T-1}(1,1,D,G) + E[PDV^z_{T-1}(1,1,D,G)]\right]^{-\gamma} \end{bmatrix}$$

Unfortunately, this is a non-linear system of equations. To find the value function in $T-2$, one could fix a grid of m_{T-2}, and for each point in the grid solve for optimal consumption and obtain the value function. Interpolation across m's would yield the value function for any m_{T-2}. Note also that the previous derivation is also valid for $0 < t < T-2$, so backward induction would allow us to unwind this problem and construct the value function in period 1. The problem in period 0 is slightly different, as the state is $(0,0)$ and wealth is $\omega + z_0(0,0,D,G) + FDA$ with certainty[28], but the same tools apply.

One issue we've abstracted away from up to now is the no-borrowing constraint: $a_{T-1} \geq 0$. Incorporating this constraint implies that when m_{T-1} is sufficiently low, consumption will not be the solution to the aforementioned problem, but rather m_{T-1} itself. This creates a discontinuity in the optimal policy function. Since our approximations to the optimal policy and value functions are constructed by interpolation, it is crucial to incorporate the point where the discontinuity takes place into the grid of points to be evaluated. This ensures that the no-borrowing constraint is properly accounted for in the model. At the point where the no-borrowing constraint binds, $\hat{m}_{T-1}$, the marginal value of consuming m_{T-1} must be equal to the marginal utility of saving 0.

We use the Endogenous Gridpoints Method (Carroll (2006)) to find the solution to the aforementioned problem. At a high level, the strategy is to solve the model for a grid of asset states, and then to interpolate across states to obtain the policy function and the value function. EGM allows us to solve the model efficiently,

[28]Recall that FDA is the free disposal amount, another attribute of an annuity offer. In most cases, it is 0.

by re-writing the problem in a way that allows us to back out a solution using an inversion rather than root-finding. The details of the implementation for $T-2$ are presented below:

Numerical Calculation of Policy Function in $T-2$:

1. Select a grid of a_{T-2} with support $[0, \bar{a}_{T-2}]$, where:

$$\bar{a}_{T-2} = R^{T-2}\omega + \sum_{\tau=0}^{T-2} R^{T-2-\tau} z_\tau(0,0,D,G) \tag{49}$$

2. Calculate the relevant quantities for the unconstrained problem:

$$m_{T-1}(d_{T-1}, q_{T-1}, D, G) = a_{T-2} \cdot R + z_{T-1}(d_{T-1}, q_{T-1}, D, G)) \tag{50}$$

$$\mathfrak{c}_{T-1}(0, q_{T-1}) = \left(\frac{R}{((\delta \cdot \beta \cdot R)^{1/\gamma} + R)} \cdot m_{T-1}(0, q_{T-1}, D, G) \right)^{-\gamma} \tag{51}$$

$$c_{T-2}(0, q_{T-2}) = \left[\delta \cdot R \cdot \Gamma_{T-2}(0, q_{T-2})' \begin{bmatrix} \mathfrak{c}_{T-1}(0,0) \\ \mathfrak{c}_{T-1}(0,1) \\ \beta \cdot [m_{T-1}(1,0,D,G)]^{-\gamma} \\ \beta \cdot [m_{T-1}(1,1,D,G)]]^{-\gamma} \end{bmatrix} \right]^{-\frac{1}{\gamma}} \tag{52}$$

$$\mathfrak{c}_{T-2}(0, q_{T-2}) = c_{T-2}(0, q_{T-2})^{-\gamma} \tag{53}$$

$$m_{T-2}(0, q_{T-2}) = c_{T-2}(0, q_{T-2}) + a_{T-2} \tag{54}$$

$$V_{T-1}(0, q_{T-1}) = \left(\frac{1 + (\delta \cdot \beta \cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma} \right) \left(\frac{R \cdot m_{T-1}(0, q_{T-1})}{(\delta \cdot \beta \cdot R)^{1/\gamma} + R} \right)^{1-\gamma} \tag{55}$$

$$V_{T-1}(1, q_{T-1}) = \beta \left(\frac{m_{T-1}(1, q_{T-1})^{1-\gamma}}{1-\gamma} \right) \tag{56}$$

$$V_{T-2}(0, q_{T-2}) = \frac{c_{T-2}(0, q_{T-2})^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_{T-2}(0, q_{T-2}) \begin{bmatrix} V_{T-1}(0,0) \\ V_{T-1}(0,1) \\ V_{T-1}(1,0) \\ V_{T-1}(1,1) \end{bmatrix} \tag{57}$$

3. Denote $\hat{m}_{T-2}(0, q_{T-2})$ the solution to equation (54) when $a_{T-2,i} = 0$. This is the lowest level of wealth

that is unconstrained. Define

$$\hat{V}_{T-1}(0,q_{T-1}) = \left(\frac{1+(\delta\cdot\beta\cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma}\right)\left(\frac{R\cdot z_{T-1}(0,q_{T-1},D,G)}{(\delta\cdot\beta\cdot R)^{1/\gamma}+R}\right)^{1-\gamma} \tag{58}$$

$$\hat{V}_{T-1}(1,q_{T-1}) = \beta\left(\frac{z_{T-1}(1,q_{T-1},D,G)^{1-\gamma}}{1-\gamma}\right) \tag{59}$$

$$\hat{\mathfrak{c}}_{T-2,j}(0,q_{T-2}) = m_{T-2,j}^{-\gamma} \tag{60}$$

$$\hat{V}_{T-2,j}(0,q_{T-2},m_{T-2}) = \frac{m_{T-2,j}^{1-\gamma}}{1-\gamma} + \delta\cdot\Gamma_{T-2}(0,q_{T-2})\begin{bmatrix}\hat{V}_{T-1}(0,0)\\ \hat{V}_{T-1}(0,1)\\ \hat{V}_{T-1}(1,0)\\ \hat{V}_{T-1}(1,1)\end{bmatrix} \tag{61}$$

4. Use interpolation to obtain $\grave{\mathfrak{c}}_{T-2}(0,q_{T-2},m_{T-2})$, $\hat{\grave{\mathfrak{c}}}_{T-2,j}(0,q_{T-2})$, $\grave{V}_{T-2,}(0,q_{T-2},m_{T-2})$, and $\hat{\grave{V}}_{T-2,}(0,q_{T-2},m_{T-2})$ for the unconstrained problem.

5. Correct for the no-borrowing constraint by constructing a part exact, part interpolated policy and value function for this period [29]

$$\grave{\mathfrak{c}}^*_{T-2}(0,q_{T-2},m_{T-2}) = \begin{cases} m_{T-2}^{-\gamma} & \text{if } m_{T-2} < \hat{m}_{T-2}(0,q_{T-2}) \\ \grave{\mathfrak{c}}_{T-2}(0,q_{T-2},m_{T-2}) & \text{otherwise} \end{cases}$$

$$\grave{V}^*_{T-2}(0,q_{T-2},m_{T-2}) = \begin{cases} \hat{V}_{T-2,}(0,q_{T-2},m_{T-2}) & \text{if } m_{T-2} < \hat{m}_{T-2}(0,q_{T-2}) \\ \grave{V}_{T-2,}(0,q_{T-2},m_{T-2}) & \text{otherwise} \end{cases}$$

There are three issues worth discussing in this procedure: first, we assume that individuals cannot borrow against future annuity payments (the lower bound of a is 0). This is consistent with our knowledge of the Chilean banking system. Second, we set the upper bound of the support of assets as the PDV of initial wealth plus the PDV of the maximum sequence of previous annuity payments. This ensures that the grid of a's spans the optimal asset value in $T-2$, as in the model the agent cannot accumulate more wealth than this value. Third, we interpolate over $\mathfrak{c}(\cdot)$ instead of $c(\cdot)$. This is suggested by Carroll (2011), as the function that enters into the recursion in earlier periods is $\mathfrak{c}(\cdot)$, and not $c(\cdot)$. One could interpolate over $c(\cdot)$, and then raise the interpolated value to the power of $-\frac{1}{\gamma}$, but that is less accurate is simply interpolating over $\grave{\mathfrak{c}}$. With these objects, we can solve the problem for $T-3, T-4, ..., 0$ by recursion.

Numerical Calculation of Policy Function in t:

1. Select a grid of a_t with support $[0,\bar{a}_t]$:

$$\bar{a}_t = R^t\omega + \sum_{\tau=0}^{t} R^{t-\tau} z_\tau(0,0,D,G) \tag{62}$$

[29] Note that the solution objects for the $T-2$ problem are exact when the constraint binds.

2. Calculate the relevant quantities for the unconstrained problem (suppressing the dependence on D and G to simplify notation):

$$m_{t+1}(0,q_{t+1}) = a_t \cdot R + z_{t+1}(0,q_{t+1}) \tag{63}$$

$$c_t(0,q_t) = \left[\delta \cdot R \cdot \Gamma_t(0,q_t)' \begin{bmatrix} \grave{\mathfrak{c}}^*_{t+1}(0,0,m_{t+1}(0,0)) \\ \grave{\mathfrak{c}}^*_{t+1}(0,1,m_{t+1}(0,0)) \\ \beta \cdot \left[m_{t+1}(1,0) + E[PDV^z_{t+1}(1,0,D,G)]\right]^{-\gamma} \\ \beta \cdot \left[m_{t+1}(1,1) + E[PDV^z_{t+1}(1,1,D,G)]\right]^{-\gamma} \end{bmatrix} \right]^{-\frac{1}{\gamma}} \tag{64}$$

$$\mathfrak{c}_t(0,q_{T-2}) = c_t(0,q_{T-2})^{-\gamma} \tag{65}$$

$$m_t(0,q_t) = c_t(0,q_t) + a_t \tag{66}$$

$$V_{t+1}(1,q_{t+1}) = \beta \left(\frac{m_{t+1}(1,q_{t+1})^{1-\gamma}}{1-\gamma} \right) \tag{67}$$

$$V_t(0,q_t) = \frac{c_t(0,q_t)^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_t(0,q_t) \begin{bmatrix} \grave{V}^*_{t+1}(0,0,m_{t+1}) \\ \grave{V}^*_{t+1}(0,1,m_{t+1}) \\ V_{t+1}(1,0) \\ V_{t+1}(1,1) \end{bmatrix} \tag{68}$$

3. Define $\hat{m}_t(0,q_t)$ as the level of wealth obtained at $a_t = 0$ and

$$\hat{V}_{t+1}(1,q_{t+1}) = \beta \left(\frac{E[PDV^z_{t+1}(1,q_{t+1},D,G)]^{1-\gamma}}{1-\gamma} \right) \tag{69}$$

$$\hat{V}_t(0,q_t) = \frac{\hat{m}_t(0,q_t)^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma_{T-2}(0,q_{T-2}) \begin{bmatrix} \grave{V}^*_{t+1}(0,0,z_{t+1}(0,0)) \\ \grave{V}^*_{t+1}(0,1,z_{t+1}(0,1)) \\ \hat{V}_{t+1}(1,0) \\ \hat{V}_{t+1}(1,1) \end{bmatrix} \tag{70}$$

4. Use interpolation to obtain $\grave{\mathfrak{c}}_t(0,q_t,m_t)$, $\hat{\grave{\mathfrak{c}}}_{t,j}(0,q_t)$, $\grave{V}_{t,}(0,q_t,m_t)$, and $\hat{\grave{V}}_{t,}(0,q_t,m_t)$ for the unconstrained problem.

5. Correct for the no-borrowing constraint:

$$\grave{\mathfrak{c}}^*_t(0,q_t,m_t) = \begin{cases} m_t^{-\gamma} & \text{if } m_t < \hat{m}_t(0,q_{t+1}) \\ \grave{\mathfrak{c}}_t(0,q_t,m_t) & \text{otherwise} \end{cases}$$

$$\grave{V}^*_t(0,q_t,m_t) = \begin{cases} \hat{V}_t(0,q_t,m_t) & \text{if } m_t < \hat{m}_t(0,q_t) \\ \grave{V}_t(0,q_t,m_t) & \text{otherwise} \end{cases}$$

6. Repeat for $t-1$

Note that again, the constrained segment requires no additional interpolation and hence its implementation is both efficient and precise.We can recover the object of interest (the value of an annuity offer: $V(0,0,\omega_i,D,G)$) after the $t=0$ step in the previous recursion.

C.4 Computation of the Solution to the PW Problem

In period T, $\mu_T = 1$ and $PW_T = 0$, so $m_T = a_{T-1} \cdot R$ and:

$$V_T(0,m_T,PW_T) = \beta \cdot \frac{m_T^{1-\gamma}}{1-\gamma} \tag{71}$$

Then in the next-to-last period:

$$V_{T-1}(0,m_{T-1},PW_{T-1}) = \max_{c_{T-1}} \frac{c_{T-1}^{1-\gamma}}{1-\gamma} + \frac{\delta \cdot \beta}{1-\gamma}((m_{T-1}-c_{T-1}) \cdot R)^{1-\gamma} \tag{72}$$

The optimal policy and value functions in $T-1$ are then:

$$c_{T-1}(m_{T-1}) = \frac{R}{((\delta \cdot \beta \cdot R)^{\frac{1}{\gamma}} + R)} \cdot m_{T-1} \tag{73}$$

$$V_{T-1}(0,m_{T-1}) = \left(\frac{1+(\delta \cdot \beta \cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma} \right) \left(\frac{R \cdot m_{T-1}}{(\delta \cdot \beta \cdot R)^{1/\gamma} + R} \right)^{1-\gamma} \tag{74}$$

Note that, conditional on m_{T-1}, there is no dependence on PW_{T-1}. This is because PW_{T-1} will shift m_{T-1}, as $m_{T-1} = a_{T-2} \cdot R + z_{T-1}(PW_{T-1},a)$, but conditional on m_{T-1} it becomes irrelevant. Additionally, as in the annuity problem, as long as the bequest motive is not negative the unconstrained maximizer satisfies the no-borrowing constraint.

Having solved for all the relevant quantities in $T-1$ and T, we can proceed to solve the problem in $T-2$. There are a few additional objects that need to be introduced before proceeding. First, take K draws from the distribution of R^{PW}. Each draw will be denoted by k, and draws will be held fixed across time periods. Define $\bar{R}_K$ as the largest draw from the distribution of R^{PW}. Second, define the upper bound of the grid of PW, $P\bar{W}$, recursively:

$$\begin{aligned} P\bar{W}_1 &= \bar{R}_K \cdot (PW_0 - z_t(PW_0)) \\ P\bar{W}_t &= \bar{R}_K \cdot (P\bar{W}_{t-1} - z_t(P\bar{W}_{t-1})) \end{aligned} \tag{75}$$

Finally, define the upper bound of the grid of accumulated assets as:

$$\bar{a}_t = R^t \omega + \sum_{\tau=0}^{t} R^{t-\tau} z(P\bar{W}_\tau, 0, f) \tag{76}$$

Numerical Calculation of Policy Function in $T-2$:

1. Select a grid of $(a_{T-2,i}, PW_{T-2,i})$ with support $[0, \bar{a}_{T-2}] \times [0, P\bar{W}_{T-2}]$.

2. Calculate the relevant quantities for the unconstrained problem:

$$m_{T-1,k}(0) = a_{T-2} \cdot R + z_{T-1}(R_k^{PW} \cdot (PW_{T-2} - z(PW_{T-2}), 0, a) \tag{77}$$

$$m_{T-1,k}(1) = a_{T-2} \cdot R + R_k^{PW} \cdot (PW_{T-2} - z(PW_{T-2})) \tag{78}$$

$$E_{T-2}[\mathfrak{c}_{T-1}] = \frac{1}{K} \sum_{k=1}^{K} [c_{T-1}(m_{T-1,k}(0))]^{-\gamma} \tag{79}$$

$$E_{T-2}[V'_{T-1}(1)] = \frac{\beta}{K} \sum_{k=1}^{K} [m_{T-1,k}(1)]^{-\gamma} \tag{80}$$

$$c_{T-2} = \left[\delta \cdot R \cdot \Gamma'_{T-2} \begin{bmatrix} E_{T-2}[\mathfrak{c}_{T-1}] \\ E_{T-2}[V'_{T-1}(1)] \end{bmatrix} \right]^{-\frac{1}{\gamma}} \tag{81}$$

$$m_{T-2} = c_{T-2} + a_{T-2} \tag{82}$$

$$\mathfrak{c}_{T-2} = c_{T-2}^{-\gamma} \tag{83}$$

$$E_{T-2}[V_{T-1}(0)] = \left(\frac{1 + (\delta \cdot \beta \cdot R^{1-\gamma})^{1/\gamma}}{1-\gamma} \right) \cdot \left(\frac{R}{(\delta \cdot \beta \cdot R)^{1/\gamma} + R} \right)^{1-\gamma} \frac{1}{K} \sum_{k=1}^{K} [m_{T-1,k}(0)]^{1-\gamma} \tag{84}$$

$$E_{T-2}[V_{T-1}(1)] = \frac{\beta}{1-\gamma} \cdot \frac{1}{K} \sum_{k=1}^{K} [m_{T-1,k}(1)]^{1-\gamma} \tag{85}$$

$$V_{T-2} = \frac{c_{T-2}^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma'_{T-2} \begin{bmatrix} E_{T-2}[V_{T-1}(0)] \\ E_{T-2}[V_{T-1}(1)] \end{bmatrix} \tag{86}$$

3. Denote $\hat{m}_{T-2}(PW_{T-2})$ the solution to (82) when $a_{T-2} = 0$ and the PW balance is PW_{T-2} define

$$\hat{V}_{T-2}(m_{T-2}, PW_{T-2}) = \frac{m_{T-2}^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma'_{T-2} \begin{bmatrix} E_{T-2}[V_{T-1}(0)] \\ E_{T-2}[V_{T-1}(1)] \end{bmatrix} \tag{87}$$

with the value of V_{T-1} determined by $a_{T-2} = 0$.

4. Use interpolation to obtain $\grave{\mathfrak{c}}_{T-2}(m_{T-2}, PW_{T-2})$ and $\grave{V}_{T-2}(0, m_{T-2}, PW_{T-2})$ for the unconstrained problem. Form the boundary interpolator $\grave{\hat{m}}_{T-2}(PW_{T-2})$ which determines the minimum level of unconstrained wealth for each value of the PW balance.

5. Correct for the no-borrowing constraint by constructing a part exact, part interpolated policy and value function for this period [30]

$$\grave{\mathfrak{c}}^*_{T-2}(m_{T-2}, PW_{T-2}) = \begin{cases} m_{T-2}^{-\gamma} & \text{if } m_{T-2} < \grave{\hat{m}}_{T-2}(PW_{T-2}) \\ \grave{\mathfrak{c}}_{T-2}(m_{T-2}, PW_{T-2}) & \text{otherwise} \end{cases}$$

[30] Note that the solution objects for the $T-2$ problem are exact when the constraint binds.

$$\grave{V}^{*}_{T-2}(m_{T-2},PW_{T-2}) = \begin{cases} \hat{V}_{T-2,}(m_{T-2},PW_{T-2}) & \text{if } m_{T-2} < \grave{\hat{m}}_{T-2}(PW_{T-2}) \\ \grave{V}_{T-2,}(m_{T-2},PW_{T-2}) & \text{otherwise} \end{cases}$$

Armed with these objects, we can solve the problem for $T-3, T-4, ..., 0$ by recursion.

<u>Numerical Calculation of Policy Function in t:</u>

1. Select a grid of (a_t, PW_t) with support $[0, \bar{a}_t] \times [0, P\bar{W}_t]$.

2. Calculate the relevant quantities for the unconstrained problem:

$$m_{t+1,k}(0) = a_t \cdot R + z_{t+1}(R_k^{PW} \cdot (PW_t - z_t(PW_t)), 0, a) \tag{88}$$

$$m_{t+1,k}(1) = a_t \cdot R + R_k^{PW} \cdot (PW_t - z_t(PW_t)) \tag{89}$$

$$E_t[\mathfrak{c}_{t+1}] = \frac{1}{K}\sum_{k=1}^{K} \grave{\mathfrak{c}}_{t+1}(m_{t+1,k}(0), PW_{t+1,k}) \tag{90}$$

$$E_t[V'_{t+1}(1)] = \frac{\beta}{K}\sum_{k=1}^{K} [m_{t+1,k}(1)]^{-\gamma} \tag{91}$$

$$c_t = \left[\delta \cdot R \cdot \Gamma'_t \begin{bmatrix} E_t[\mathfrak{c}_{t+1}] \\ E_t[V'_{t+1}(1)] \end{bmatrix}\right]^{-\frac{1}{\gamma}} \tag{92}$$

$$m_t = c_t + a_{t,i} \tag{93}$$

$$\mathfrak{c}_t = c_t^{-\gamma} \tag{94}$$

$$E_t[V_{t+1}(0)] = \frac{1}{K}\sum_{k=1}^{K} \grave{V}(0, m_{t+1,k}(0), R_k^{PW} \cdot (PW_t - z(PW_t))) \tag{95}$$

$$E_t[V_{t+1}(1)] = \frac{\beta}{1-\gamma} \cdot \frac{1}{K}\sum_{k=1}^{K} [m_{t+1,k}(1)]^{1-\gamma} \tag{96}$$

$$V_t = \frac{c_t^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma'_t \begin{bmatrix} E_t[V_{t+1}(0)] \\ E_t[V_{t+1}(1)] \end{bmatrix} \tag{97}$$

3. Denote $\hat{m}_t(PW_t)$ the solution when $a_t = 0$ and the PW balance is PW_t define

$$\hat{V}_t(m_t, PW_t) = \frac{m_t^{1-\gamma}}{1-\gamma} + \delta \cdot \Gamma'_t \begin{bmatrix} E_t[V_{t+1}(0)] \\ E_t[V_{t+1}(1)] \end{bmatrix} \tag{98}$$

   with the value of V_{t+1} determined by $a_t = 0$.

4. Use interpolation to obtain $\grave{\mathfrak{c}}_t(m_t, PW_t)$ and $\grave{V}_t(0, m_t, PW_t)$ for the unconstrained problem. Form the boundary interpolator $\grave{\hat{m}}_t(PW_t)$ which determines the minimum level of unconstrained wealth for each value of the PW balance.

5. Correct for the no-borrowing constraint by constructing a part exact part interpolated policy and value function for this period:

$$\grave{\mathfrak{c}}_t^*(m_t, PW_t) = \begin{cases} m_t^{-\gamma} & \text{if } m_t < \grave{\hat{m}}_t(PW_t) \\ \grave{\mathfrak{c}}_t(m_t, PW_t) & \text{otherwise} \end{cases}$$

$$\grave{V}_t^*(m_t, PW_t) = \begin{cases} \hat{V}_{t,}(m_t, PW_t) & \text{if } m_t < \grave{\hat{m}}_t(PW_t) \\ \grave{V}_{t,}(m_t, PW_t) & \text{otherwise} \end{cases}$$

6. Repeat for $t-1$

We can recover the object of interest (the value of a PW offer: $V_0(0, \omega, PW_0)$) after the $t = 0$ step in the previous recursion.